A simple expression for the bulk field capacity of a sloping soil horizon

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Abstract:

Field capacity is a commonly used soil parameter in surface water hydrological models, loosely defined as the moisture content of a soil after drainage. The most commonly applied expression for field capacity is defined as the remaining water in a vertical soil column subject to 1/3 atm. of pressure head. While this quantification is sufficient in some cases, the definition is not consistent with the use of bulk field capacity in calculations of lateral drainage from hillslopes, as required by some surface soil parameterizations, nor does it address additional complications arising from differences in soil texture or sample size. Here, a simple alternative expression for bulk field capacity in a sloping or vertical soil is derived directly from Richards equation with the use of the Brooks-Corey characteristics. It is demonstrated that this expression is consistent with data acquired from vertical soil columns, but may be extended to additional situations commonly found in surface water models and land surface schemes. The calculation of bulk field capacity requires only the Brooks-Corey pore size distribution index, soil air-entry pressure, and hillslope length and slope, and may be considered a physically based alternative to pedotransfer function or lookup table approaches. Copyright © 2010 John Wiley & Sons Ltd and Crown in the right of Canada.

KEY WORDS   field capacity; retained soil moisture; hillslope hydrology; analytical solution

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INTRODUCTION

Field capacity is loosely defined as the amount of water remaining in a drying soil after excess moisture has drained from its pores and the rate of drainage from the soil is negligible (Veihmeyer and Hendrickson, 1931). It is used as an input to a variety of hydrological process submodels (e.g. those for soil evaporation (Dyck, 1983), bypass flow (Federer, 2002), infiltration, etc.). It is also used for calculation of common metrics in soil physics, for example, available water capacity (Salter and Haworth, 1961). Often treated as a constant for a given soil, field capacity may be measured as a point value in the field by monitoring the soil moisture in a covered soil initially at saturation until the flow rate, approximately reaches an asymptotic value, typically 2 or 3 days after a wetting event. Unfortunately, this definition is insufficient for regional watershed models, where field data are typically not available, and bulk or representative values of field capacity may be needed. Quantification is confounded by the fact that field capacity is dependent upon the soil characteristics, wetting history, suction pressure applied to the element and the means of defining a 'negligible' drainage rate.

The classical quantitative definition of field capacity relies upon the empirically derived suction of -0.33

bar (340 cm) of pressure head, as determined via correlation with observations at the point scale (Richards and Weaver, 1944). While this definition has been maintained for many lookup-table based hydrological models, other researchers have since used alternative forms chosen to reduce the number of specified parameters in distributed hydrological models (Nachabe, 1998; Federer, 2002; Verseghy, 2009). These alternate forms require the use of a specified limiting flux (Nachabe, 1998), specified limiting conductivity (Verseghy, 2009), or specified combination of depth, gravity potential gradient and time period (Federer, 2002) that corresponds to the negligible drainage rate. The primary limitation of all these methods is that they rely upon either empirical values for the limiting suction head, flux, gradient, or conductivity and therefore introduce additional parameters to typically over-parameterized hydrological models. In addition, these expressions are specific to field capacity due solely to vertical drainage. Lastly, a number of pedotransfer functions (Gupta and Larson, 1979; Bell and van Keulen, 1996) have been derived for field capacity. These pedotransfer functions, while useful, tend to be location-specific and are in any case developed from the 1/3 bar approximation, and therefore suffer from the same questions about the physical basis of their applicability. They, too, cannot be extended to the hillslope case.

Here, an alternative physically based expression for field capacity is defined that is consistent with field measurements and existing pedotransfer functions, and may further be extended to the lateral field capacity concept

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for use in land surface schemes that characterize a watershed as a set of sloping soil units (Pietroniro *et al.*, 2007). This expression, based on an approximate physical solution to the one-dimensional Richards equation, includes only standard measurable soil parameters, thus making it appealing for improving the parsimony of a given surface water model.

SOLUTION

The sloping soil horizon is assumed to be homogeneous and subject only to lateral drainage through a downhill seepage face, as depicted in Figure 1a. Flow is assumed to occur only in the downhill direction, which leads to the following form of the 1D Richards equation:

$$\theta_s \frac{\partial S}{\partial t} = \frac{\partial}{\partial X}\left(k(S)\left(\frac{\partial \psi}{\partial X} + \Delta\right)\right) \quad (1)$$

where $\Delta = \frac{\partial z}{\partial X} = -\alpha/\sqrt{1+\alpha^2}$, and α is the slope of the soil horizon. Note that $\Delta = 1$ for vertical infiltration into flat ground (as in Figure 1b) and that residual water content is assumed to be negligible.

The soil characteristics are assumed to be well-characterized by the Brooks and Corey (1964) model for the interrelationship between saturation, pressure head, and unsaturated hydraulic conductivity, that is,

$$\psi = \psi_a S^{-b} \quad (2)$$

where ψ [L] is the matric, or suction, pressure, ψ_a [L] is the air-entry pressure, S [−] is the saturation of the soil and b is an empirically defined fitting parameter [−], assumed to be a constant for the soil (b is the inverse of the pore size distribution index). Also,

$$k = k_s S^c \quad (3)$$

where k [L/T] is the hydraulic conductivity, k_s [L/T] is the saturated hydraulic conductivity, and c is the pore disconnectivity index approximated by $3b+2$. The Brooks–Corey exponents are readily estimated from soil textures using empirical-regression relationships (Rawls *et al.*, 1982; Saxton *et al.*, 1986) or a lookup table approach.

The approach taken here is to divide the problem domain into a downhill zone where gravity effects are dominant (the gradient of soil suction is negligible) and an uphill zone where suction effects are dominant (the total head gradient is negligible). The two zones are separated by a moving front, $X_f(t)$ [L]. Bulk field capacity is then defined as the bulk saturation when the moving front reaches the seepage face. Because the head gradient (and therefore the flux) behind this front is effectively zero, this definition is a means of mathematically formalizing the negligible drainage condition.

First, the problem is solved for the entire domain using the assumption that the variation of matrix potential is small compared with the slope (i.e. $\frac{\partial \psi}{\partial X} \ll \frac{\partial z}{\partial X}$). With a negligible suction gradient, Equation (1) simplifies to:

$$\frac{\partial S}{\partial t} = \frac{\Delta}{\theta_s}\frac{\partial k(S)}{\partial X} \quad (4)$$

Substituting the Brooks–Corey relationship (Equation (3)),

$$\frac{\partial S}{\partial t} = \frac{\beta}{c}\frac{\partial S^c}{\partial X} = \beta S^{c-1}\frac{\partial S}{\partial X} \quad (5)$$

where

$$\beta = \frac{k_s c \Delta}{\theta_s} \quad (6)$$

Using the implicit function theorem, the problem may be reformulated in terms of the horizontal velocity and solved for $S(X, t)$,

$$S(X, t) = \left[\frac{1}{\beta}\frac{X - X_0}{t}\right]^{\frac{1}{c-1}} \quad (7)$$

where X_0 [L] is an arbitrary constant. The choice of $X_0 = 0$ corresponds to an initial condition of $S(X, 0) \geq 1$, that is, the top of the hillslope has a non-zero (actually infinite) saturation, thus the initial condition of a fully saturated domain is met. It may be verified that the parameter β is actually the celerity of the saturated/unsaturated interface. Note that the domain of the mathematical solution to this equation is limited to saturations less than 1, so in practice the kinematic wave solution is re-expressed as:

$$S^+(X, t) = \min\left(\left[\frac{1}{\beta}\frac{X}{t}\right]^{\frac{1}{c-1}}, 1\right) \quad (8)$$
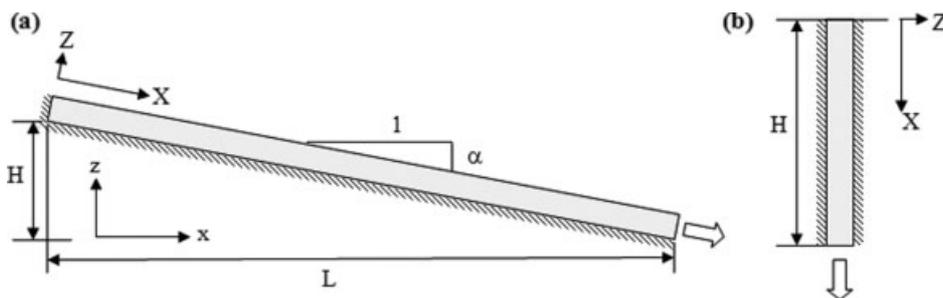

Figure 1. The conceptual model of a sloping soil horizon (a) initially at saturation and allowed to drain to field capacity. Vertical infiltration into flat ground (b) may be simulated by taking the limit as α goes to infinity

where the $^+$ superscript is used to distinguish the gravity-dominated (downhill) portion of the solution from the uphill portion ($^-$ superscripts). Note that this is an exact solution to the simplified Richards equation, provided the suction head gradient is small compared with the slope of the hillside.

For the uphill zone, the total head gradient is assumed negligible (i.e. the flow rate in the uphill zone is nearly zero), providing the following linear solution for suction head:

$$\psi^- \approx \psi_f - \Delta(X - X_f) \qquad (9)$$

where ψ_f [L] is the suction head at the interface between solutions (i.e. at X_f). At the zonal intersection ($X_f(t)$), continuity in both saturation and the derivative of saturation is assumed. Note that through the relationship of Equation (2), continuity of suction and its derivative are implicitly preserved. Also, although the gravity-dominated solution is based upon the assumption of a negligible suction derivative, because the solution is obtained purely in terms of saturation, this derivative is actually non-zero in the solution. This provides a means to solve for ψ_f (the other unknown, X_f, is not explicitly required for this analysis, though may also be obtained). Applying continuity of suction gradient:

$$\left.\frac{\partial\psi^+}{\partial X}\right|_{X_f} = \left.\frac{\partial\psi^-}{\partial X}\right|_{X_f}$$

$$\psi_a\left(-\frac{b}{c-1}\right)\frac{1}{X_f}\left[\frac{1}{\beta}\frac{X_f}{t}\right]^{-\frac{b}{c-1}} = \Delta$$

$$\psi_f = -\frac{c-1}{b}\Delta X_f \qquad (10)$$

Substituting this result into Equation (9), the following expression for matric potential uphill of the zonal intersection is obtained:

$$\psi^- = \frac{\Delta}{b}(bX - (3b+2)X_f)$$

and, using Equation (2),

$$S^- = \left(\frac{b\psi_a}{\Delta(bX - (3b+2)X_f)}\right)^{1/b} \qquad (11)$$

This solution (the combination of Equations (11) and (8)) suffers from a few drawbacks with regard to general application, most notably that it does not meet global mass balance in the uphill portion of the solution owing to the relaxed assumptions about the suction gradient. Although the solution is not recommended for calculation of point saturations, the suction gradient does in fact approach the appropriate steady-state solution.

By integrating the solution along the hillslope, a useful operational definition of bulk field capacity is obtained, one which is consistent with existing literature and field measurements, as is shown in the next section. The above expression is integrated from $X = 0$ to $X_f = L\sqrt{1+\alpha^2}$ (the length of the hillslope) to obtain the limiting bulk saturation at which the total head gradient is everywhere

negligible (i.e. the point in time at which drainage ceases):

$$\overline{\theta}_{fc} = \frac{\theta_s}{L\sqrt{1+\alpha^2}}\int_0^{L\sqrt{1+\alpha^2}} S^-(X, X_f)\,dX$$

$$\overline{\theta}_{fc} = \frac{\theta_s}{b-1}\left(-\frac{\psi_a b}{L\Delta}\right)^{1/b}((3b+2)^a - (2b+2)^a) \qquad (12)$$

where $a = (b-1)/b$. This general operational expression for bulk field capacity, the primary contribution of this article, is examined in the next section.

The approach outlined above suggests another, potentially simpler, approach for defining field capacity: taking the simple steady-state solution (linear in ψ), and integrating over the modeled domain to obtain the bulk field capacity. This alternative approach will grossly underestimate field capacity because it presumes an infinite time during which drainage occurs. In contrast, the approach outlined above provides drainage durations that are consistent with the practical definition of field capacity. Using texturally aggregated data from Clapp and Hornberger (1978), time to reach field capacity using Equation (12) is on the order of days, ranging from 1·6 days for silt loam to 12·3 days for clay, commensurate with field observations.

It is important to note that this approach is based upon a number of critical assumptions which help determine the context in which its use is appropriate. Because flow is assumed to be one-dimensional, situations where two-dimensional effects are dominant may exhibit different drainage characteristics. Therefore, the method should only be used for vertical drainage into soil or interflow along shallow soils either underlain by bedrock or with a conductivity that decreases with depth. Likewise, the assumption of homogeneity necessitates caution when using the definition of Equation (12) for layered or highly heterogeneous soils.

TESTING

Because estimates of the effective field capacity for sloping soils are not readily available, here the closed-form expression for bulk field capacity was compared to a number of data sets for vertical draining soils (Clapp and Hornberger, 1978; Rawls *et al.*, 1982; Ritchie *et al.*, 1987). These sources are often cited in soil property lookup tables used by surface water hydrological models, often mixing and matching representative parameters from one or more sources. The modeled bulk vertical field capacity is obtained using Equation (12), recognizing that $L\Delta = H$, the sample length, in the case of purely vertical drainage.

Because none of the data sets explicitly contain both measured field capacity and the complete list of parameters needed for Equation (12), each data set was individually processed for the most appropriate comparison. For the Clapp and Hornberger (1978) data, estimated field capacity was first calculated using the standard -0.33 bar
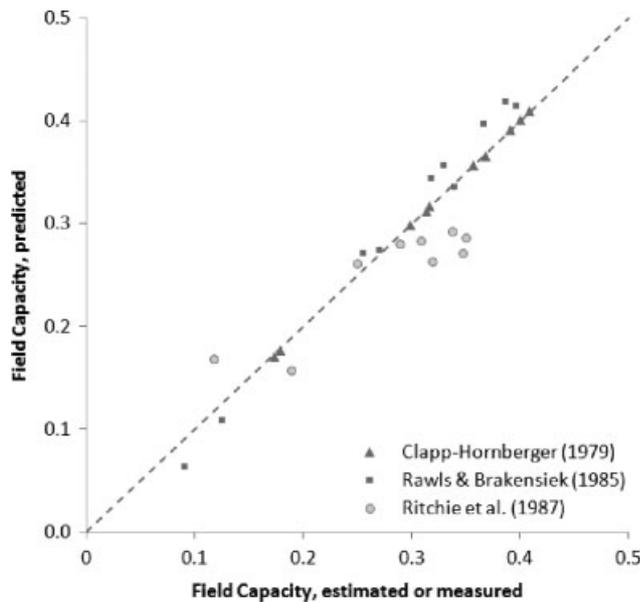
Figure 2. Comparison of the operational definition of field capacity (Equation 12) to measured and estimated field capacities of vertical soil columns. Each data point represents a group of soils under one textural class

Figure 3. Variation in bulk field capacity, as predicted with Equation (12), with increasing hillslope length (L) and/or decreasing steepness (Δ) for standard USDA soil textures

approximation $\theta_{fc} = \theta_s (340/\psi_a)^{-b}$ and compared with the results of Equation (12) using a depth, H, of 4 feet (1·22 m), as reported in (Holtan *et al*., 1968), the original source of the Clapp–Hornberger data. For the data of Rawls *et al.* (1982), a typical agricultural tillage depth of 30 cm was assumed and the reported moisture content at −0·33 bar was used as a surrogate for estimated field capacity. The Ritchie *et al.* (1987) was the lone source of measured field capacity data. For this data set, Brooks–Corey b parameters were calculated from the 2 to 4 points on the matric potential-soil moisture curves provided for each sample, then average porosity, the inverse of the pore size distribution index, b and sample length were calculated for each US Department of Agriculture (USDA) soil texture. Air-entry pressure, not reported in the Ritchie data set, was taken from the textural averages of the Clapp–Hornberger data set. Results are depicted in Figure 2.

Clearly, the approximation of Equation (12) does quite well in estimating the field capacity for vertical draining soils over a wide range of textural classes and sample sizes (from 30 to 122 cm).

EXTENSION TO SLOPING SOILS

While the vertical testing from above is not an explicit indicator that the solution derived here works equally well for sloping soils, the physical basis of the model implies that the approach could be extended to hillslope conditions. Here we provide a brief discussion of how this may be done. As described in Soulis *et al.* (2000), micro-drainage systems may be generated easily from digital elevation models and be used as a component of a larger hydrological model. The micro-drainage systems are composed of a statistical distribution of
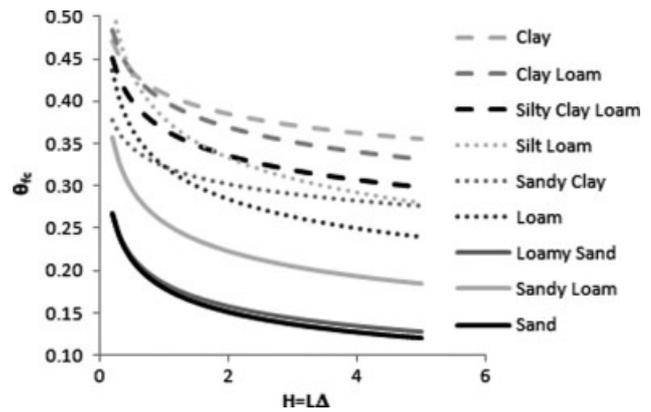
hillslopes, where lateral flow occurs both in the form of runoff (the surface component), interflow (the shallow subsurface component) and baseflow (the deep subsurface component) to a river network. Because interflow occurs along a slope, gravity effects are mitigated and effective field capacities (that would be used for calculating soil moisture fluxes) would generally be greater than that predicted from the 1/3 atm. metric. The influence of hillslope length and angle upon bulk field capacity is plotted in Figure 3 for the standard USDA soil textures (data from Clapp and Hornberger (1978)). Note that this effect is predicated upon a one-dimensional model, and is therefore most valid for shallow soils overlying an impermeable base, as found, for example, in areas where permafrost is present.

Results from Equation (12) have been directly compared with numerical solutions of the one-dimensional Richards equation (Equation (1)) for drainage from a sloping soil horizon. All boundary and initial conditions are identical to the analytical model above. The primary difference, then, between analytical and numerical solution was that the restriction of a negligible suction gradient has been removed. The finite difference equations were solved using Picard iteration, and both spatial and temporal resolution were determined to be sufficient to provide accurate results to within a moisture content of 0·001. It is important for the purposes of this comparison that there are no accepted means of defining bulk field capacity for a sloping soil even under fully controlled conditions. For the purposes of the comparison here, therefore, results of the analytical formula for bulk field capacity were directly compared with the bounding-steady state solution for a hillslope height, H, of 10 m. This approach is consistent with the practical use of bulk field capacity in hydrological simulations as a lower bound for moisture content that can be reached through interflow-based drainage alone. Similarly, the numerically calculated steady-state value is the extreme lower limit of drainage-induced moisture content.

Results of the numerical testing are depicted in Table I for a number of soil types, with soil properties and field capacities again taken from Clapp and Hornberger

Table I. Comparison between numerical results

Soiltype	Slope angle	Numerical steady-state	$\overline{\theta}_{fc}$ (Equation (12))
Sand	30	0·12	0·12
	45	0·10	0·12
	60	0·09	0·11
Loamy sand	30	0·13	0·13
	45	0·11	0·12
	60	0·10	0·12
Loam	30	0·24	0·25
	45	0·22	0·23
	60	0·19	0·23
Silty loam	30	0·30	0·28
	45	0·28	0·27
	60	0·26	0·26

(1978). It is apparent that the analytical expression produces results similar to the steady-state solution in all cases. Because the operational definition presented here is such a good approximation of the endpoint steady-state solution, it is likely the analytically derived definition of Equation (12) is best used as a lower bound for hillslope field capacity. For the purposes of simulation, a larger value will be more practical, as the steady-state limit will be reached on the order of months rather than days, as desired. The above analysis suggests that the assumption of negligible total head gradient in the suction-driven region (while quite appropriate for vertical drainage, as clear from Figure 2) is likely overly restrictive for sloping soils.

Taking into account the results from both interflow in sloping soils and vertical drainage, the use of Equation (12) is recommended for general use in infiltration (vertical) drainage models. Likewise, it provides a sensible rough estimate for drainage from sloping soils, one that is both objective and consistent with steady-state numerical solutions. However, in practice, operational field capacity for a sloping soil will be higher than the estimate provided by Equation (12). Further testing in the field is likely required for both validation and to help determine the most appropriate means of estimation.

CONCLUSION

A useful expression (Equation (12)) has been derived as an operational definition of bulk field capacity for a sloping or vertical soil column, such as used in hillslope-based hydrological models and infiltration algorithms. The expression consists of standard soil parameters required by surface water models and therefore reduces the number of calibration parameters included in these models. It has been demonstrated that the definition is consistent with the existing operational definition of field capacity for non-sloping soils, but further incorporates an dependency upon the air-entry pressure of the soil and is extendible to lateral flow calculations. The concept of bulk field capacity is shown to be dependent upon sample length, clarifying some discrepancies between reported field capacities from the literature.

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