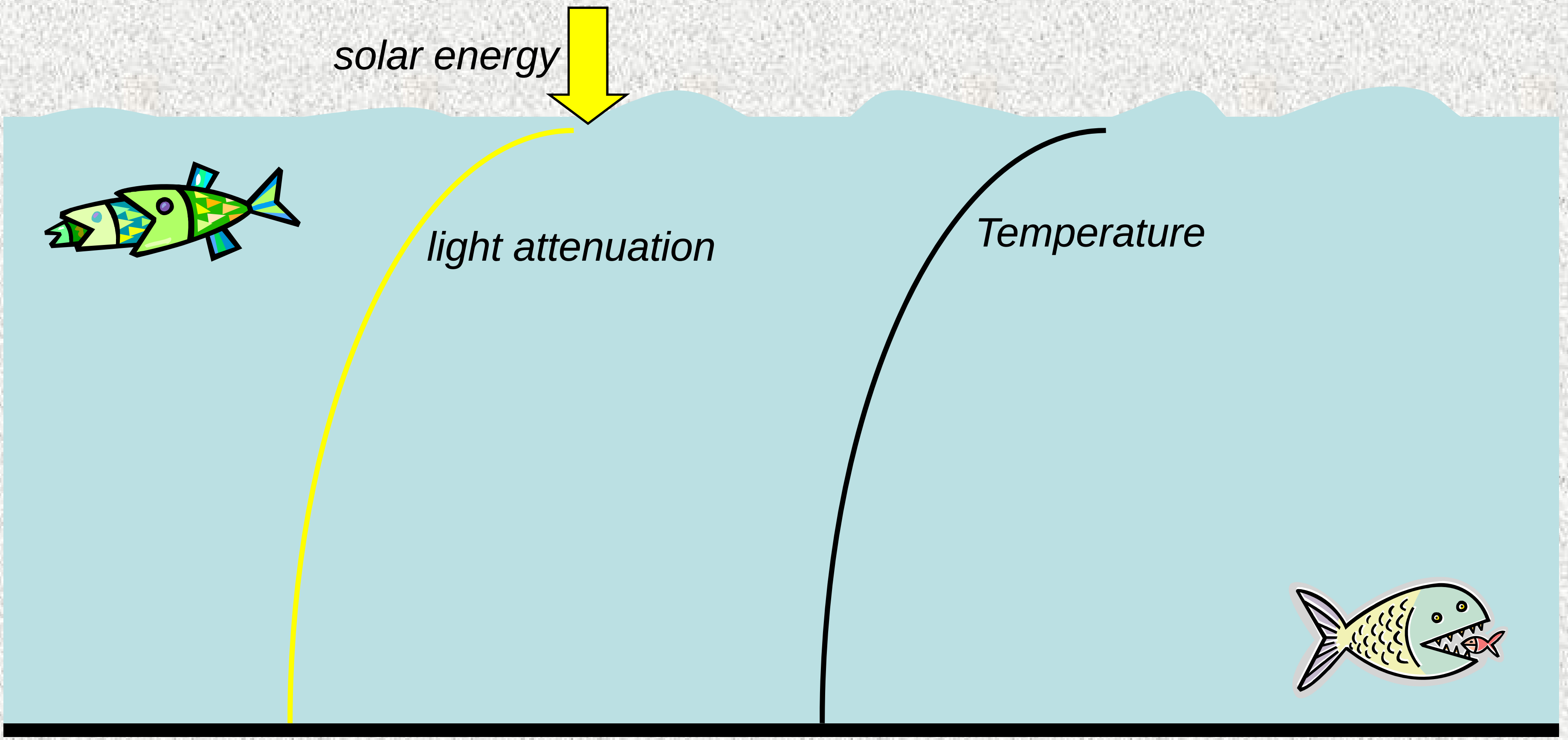
Thoughts on the Parameterization of Physical Processes

(by way of an example)

Murray D. MacKay
Environment Canada

Example: Modelling the Temperature Profile of a Lake

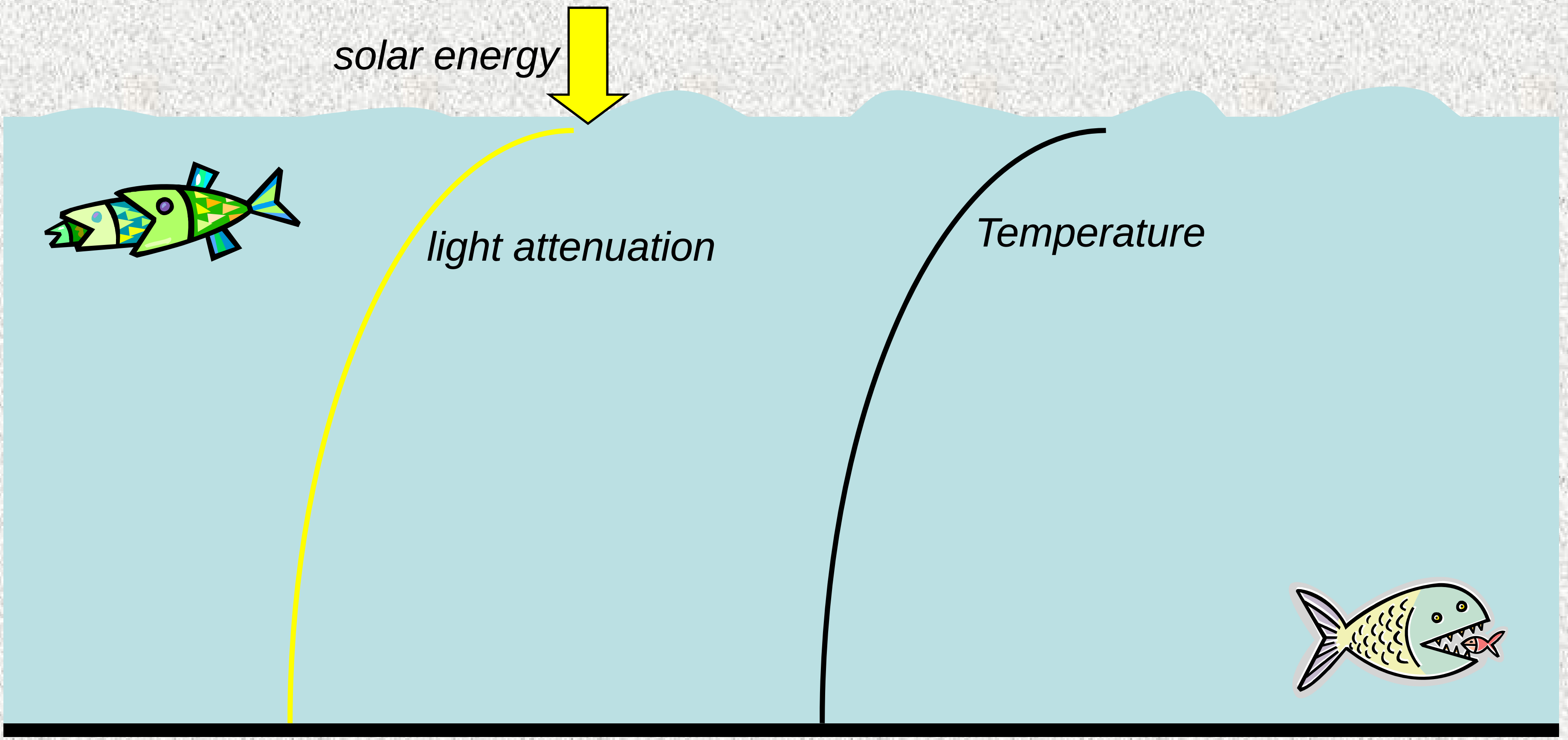


Heat Equation

$$\frac{\partial (C_w T)}{\partial t} = -\frac{\partial F}{\partial z} - \frac{\partial Q}{\partial z}$$

light extinction follows Beer’s Law

Example: Modelling the Temperature Profile of a Lake



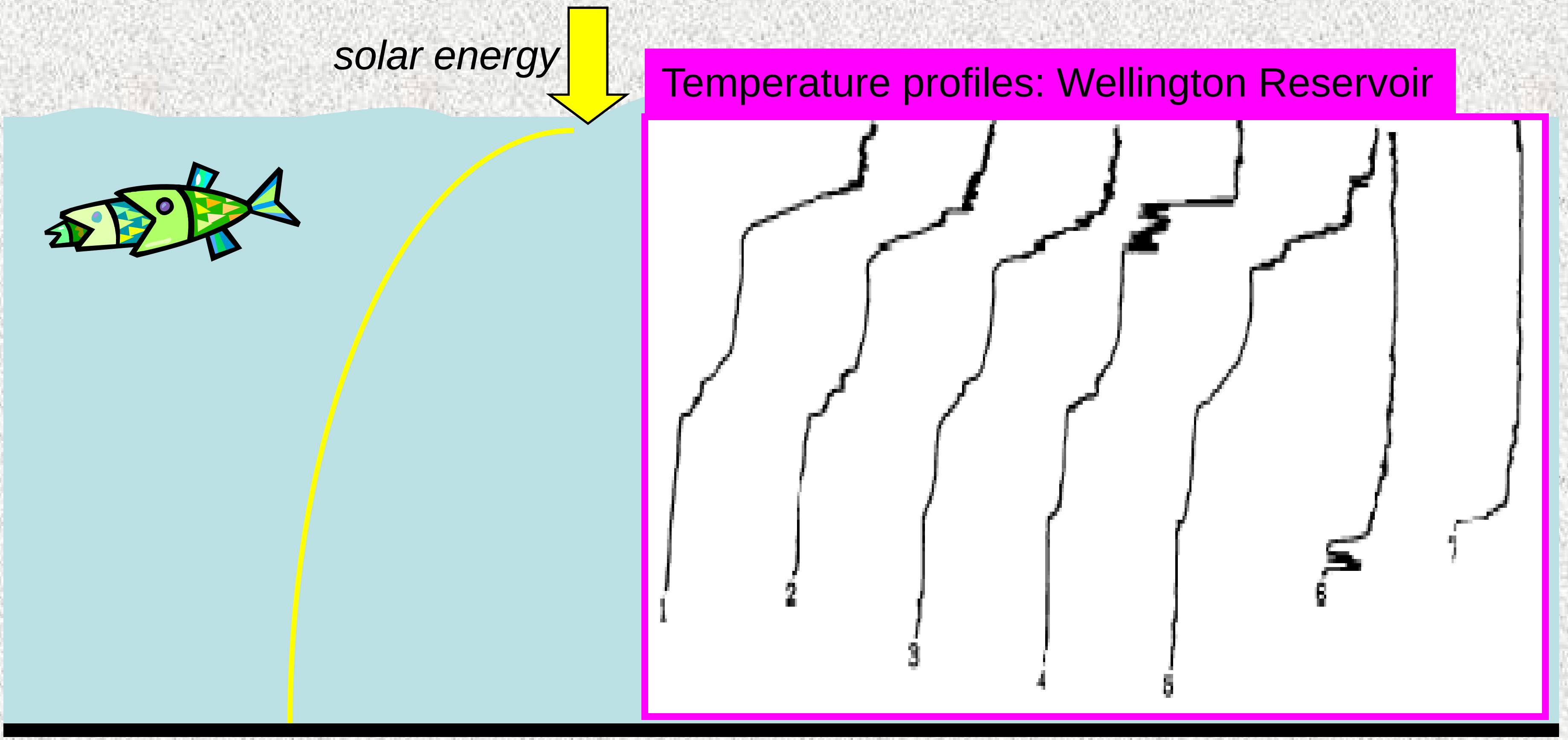
Heat Equation

$$\frac{\partial (C_w T)}{\partial t} = -\frac{\partial F}{\partial z} - \frac{\partial Q}{\partial z}$$

Fundamental Law of Heat Conduction

$$F(z) = -K_m \frac{\partial T}{\partial z}$$

Example: Modelling the Temperature Profile of a Lake



Heat Equation in a Fluid

$$\frac{\partial (C_w T)}{\partial t} = -\frac{\partial F}{\partial z} - \frac{\partial Q}{\partial z} - \frac{C_w \partial (w'T')}{\partial z}$$

$w'T'$ → turbulent fluctuations

Problem: We need to parameterize turbulence.

Approach 1: First Order Turbulence Closure

- assumes turbulent eddy motion transports heat analogously to molecular diffusion
- requires eddy size to be small
- does not parameterize turbulence itself – just the impact of turbulence on temperature profile

Heat Equation in a Fluid

$$\frac{\partial (C_w T)}{\partial t} = -\frac{\partial F}{\partial z} - \frac{\partial Q}{\partial z} - \frac{C_w \partial (w' T')}{\partial z}$$

$$\frac{\partial}{\partial z}\left(K_m \frac{\partial T}{\partial z}\right) \qquad \frac{\partial}{\partial z}\left(K_e \frac{\partial T}{\partial z}\right)$$

Property of fluid
(basically invariant)

Property of flow

K_e = constant → **Ekman Theory**

$K_e = K_e(z)$ → **constant shape**

$K_e = K_e(z,t)$ → **most general**

Henderson – Sellers (1985)

$$K_e(z,t) = \frac{\hat{k} u_* z}{P_0} \exp\left(-k^* z\right)\left[1 + 37 Ri^2\right]^{-1}$$

drift decay coefficient

$$k^* = 6.6\sqrt{\sin\vartheta}\, U^{-1.84}$$

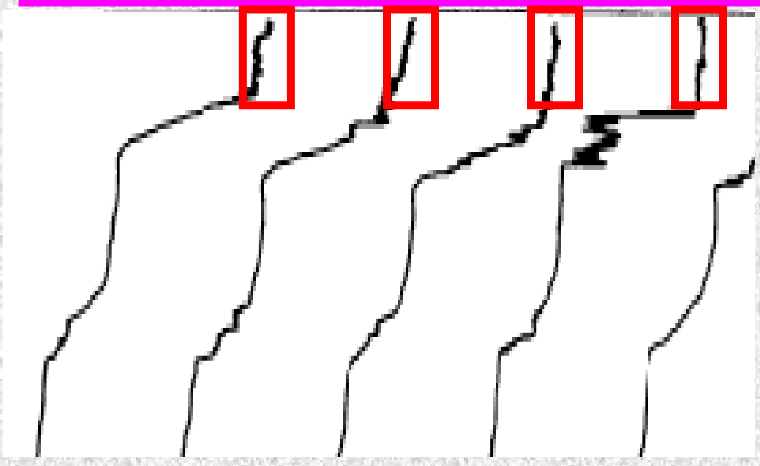
$$Ri = \frac{-1 + \left[1 + 40 N^2 \hat{k}^2 z^2 / \left(u_*^2 \exp\left(-2k^* z\right)\right)\right]^{1/2}}{20}$$

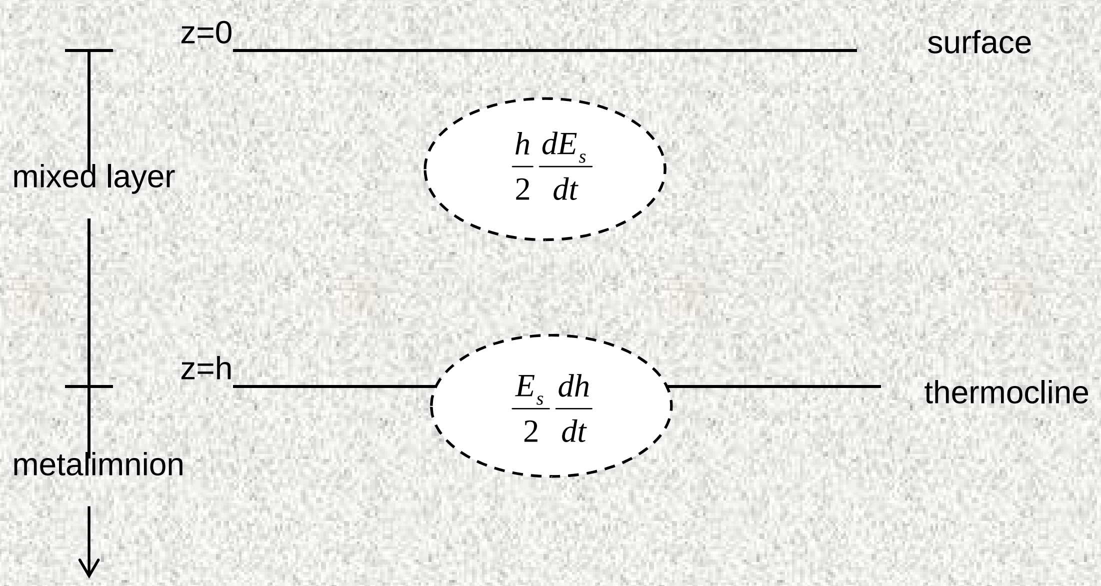
Problem: **We need to parameterize turbulence.**

Approach 2: Integrated Turbulent Kinetic Energy (TKE)

- assumes uniform well mixed layer at surface
- *does* parameterize turbulence itself as well as the impact of turbulence on temperature profile

Temperature profiles: Wellington Reservoir





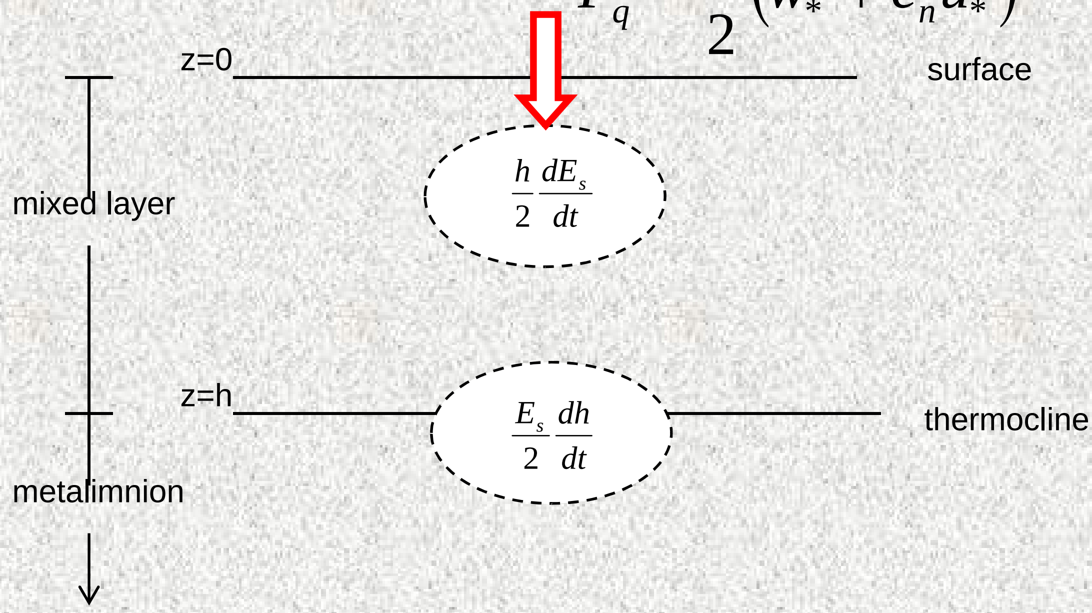
Governing Equations

$$\frac{h}{2}\frac{dE_s}{dt} = F_q - F_d - F_i$$

$$\frac{E_s}{2}\frac{dh}{dt} = F_i + F_s - F_p - F_L$$

wind and buoyancy forcing

$$F_q = \frac{1}{2}\left(w_*^3 + c_n^3 u_*^3\right)$$



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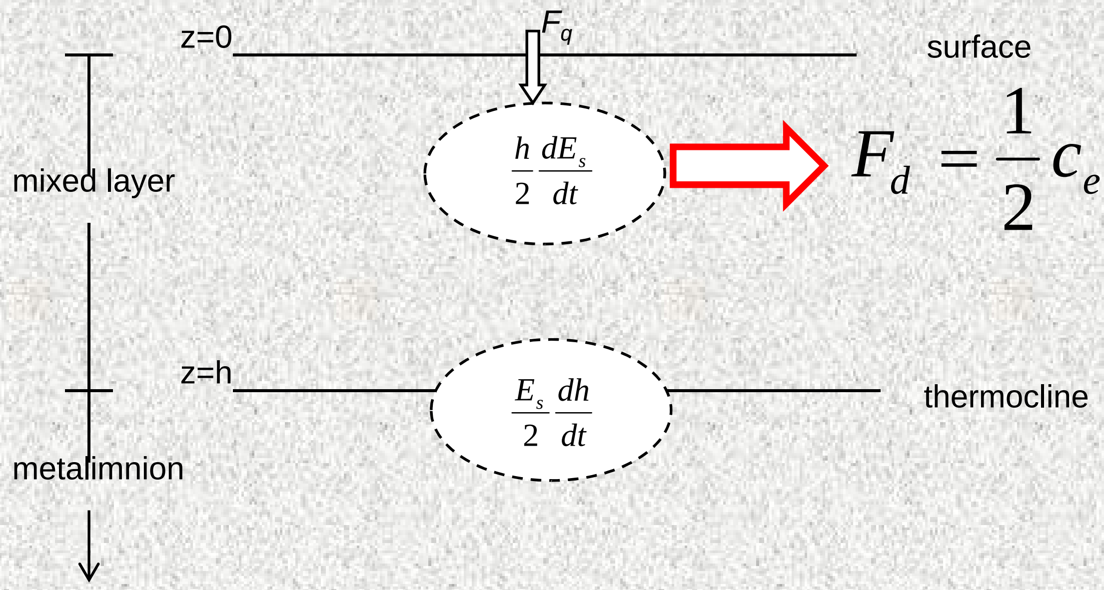
Governing Equations



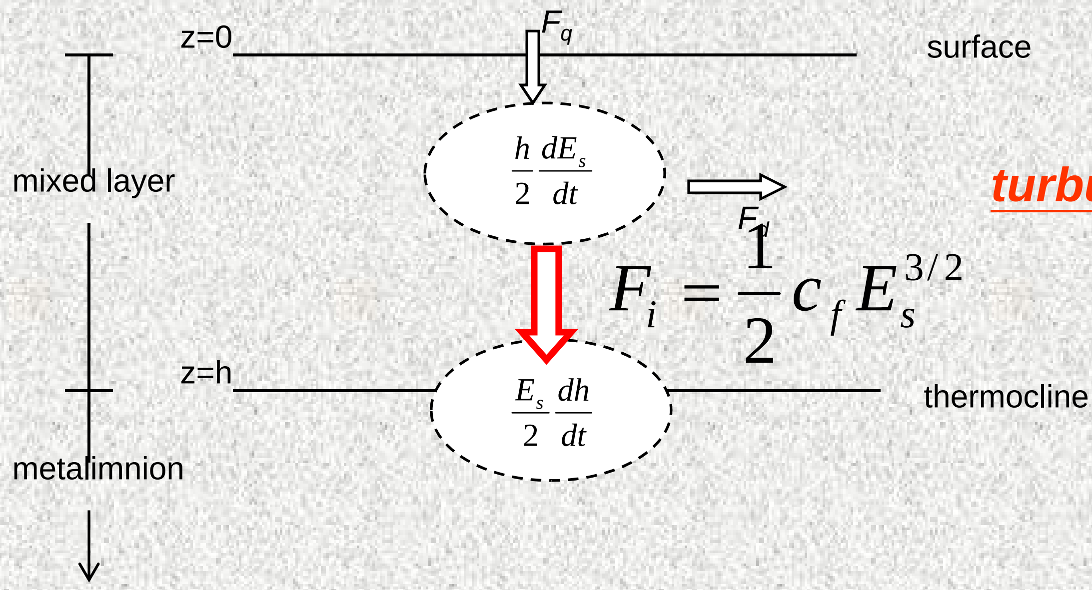
dissipation

$$F_d = \frac{1}{2} c_e E_s^{3/2}$$

Governing Equations

$$\frac{h}{2}\frac{dE_s}{dt} = F_q - F_d - F_i$$

$$\frac{E_s}{2}\frac{dh}{dt} = F_i + F_s - F_p - F_L$$



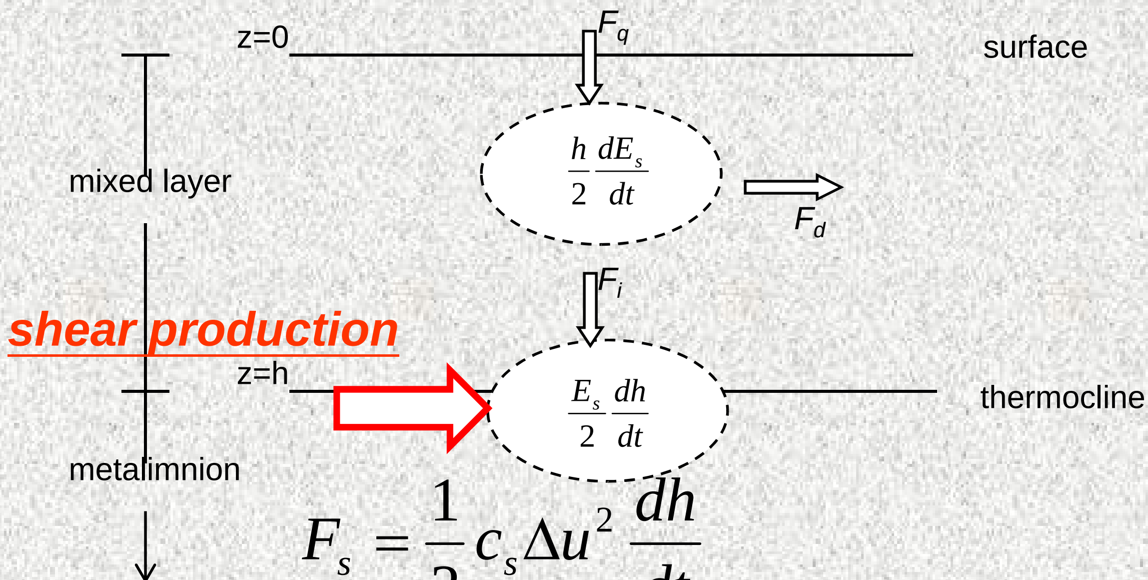
turbulent transport

$$F_i = \frac{1}{2} c_f E_s^{3/2}$$

$$\frac{h}{2}\frac{dE_s}{dt} = F_q - F_d - F_i$$

$$\frac{E_s}{2}\frac{dh}{dt} = F_i + F_s - F_p - F_L$$

Governing Equations

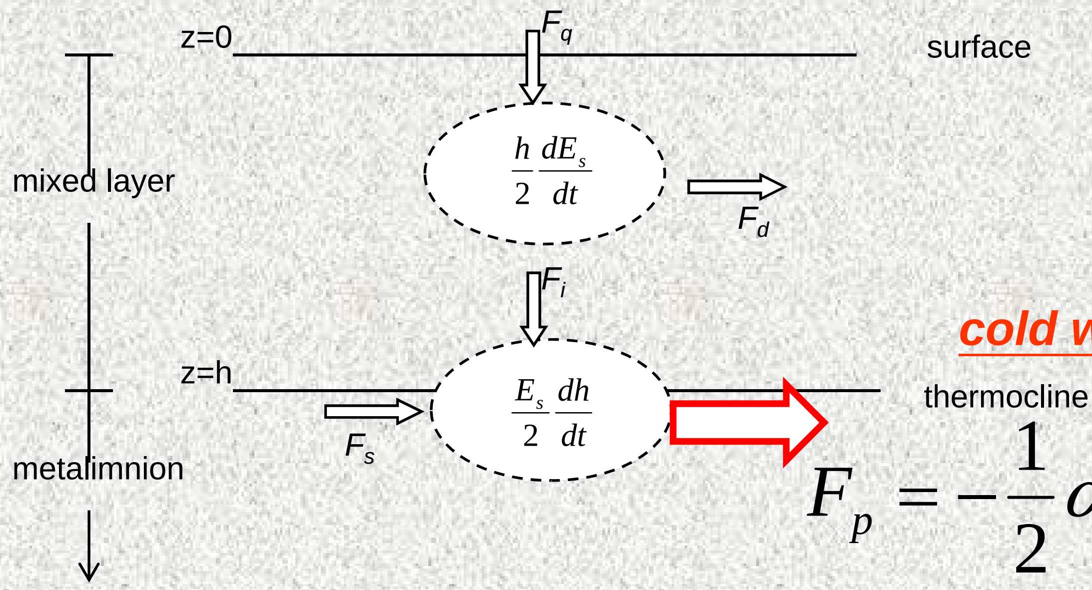


$$F_s = \frac{1}{2} c_s \Delta u^2 \frac{dh}{dt}$$

$$\frac{h}{2}\frac{dE_s}{dt} = F_q - F_d - F_i$$

$$\frac{E_s}{2}\frac{dh}{dt} = F_i + F_s - F_p - F_L$$

Governing Equations



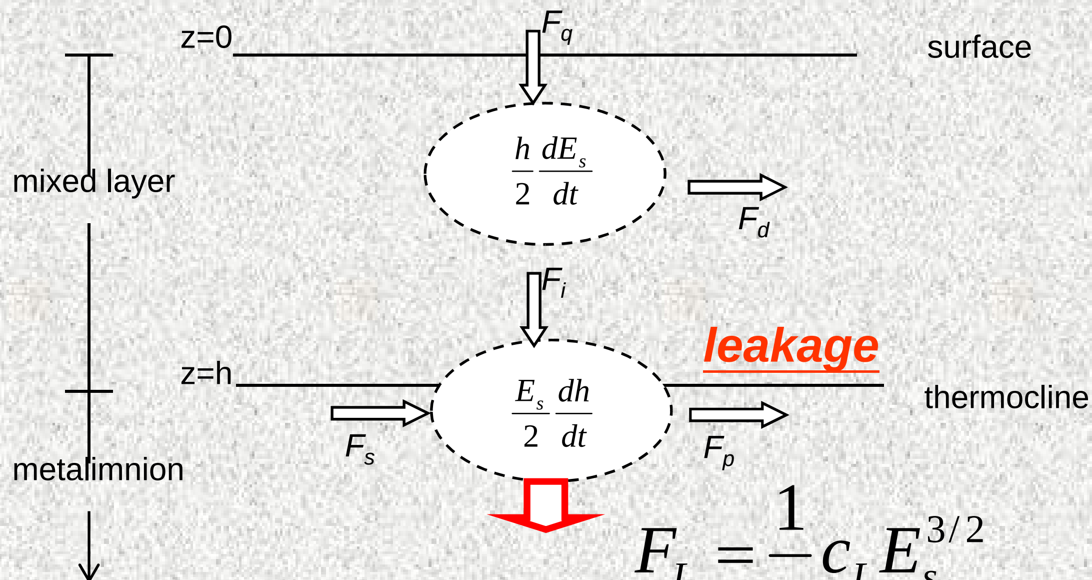
cold water entrainment

$$F_p = -\frac{1}{2}\alpha g h \Delta T \frac{dh}{dt}$$

Governing Equations

$$\frac{h}{2}\frac{dE_s}{dt} = F_q - F_d - F_i$$

$$\frac{E_s}{2}\frac{dh}{dt} = F_i + F_s - F_p - F_L$$

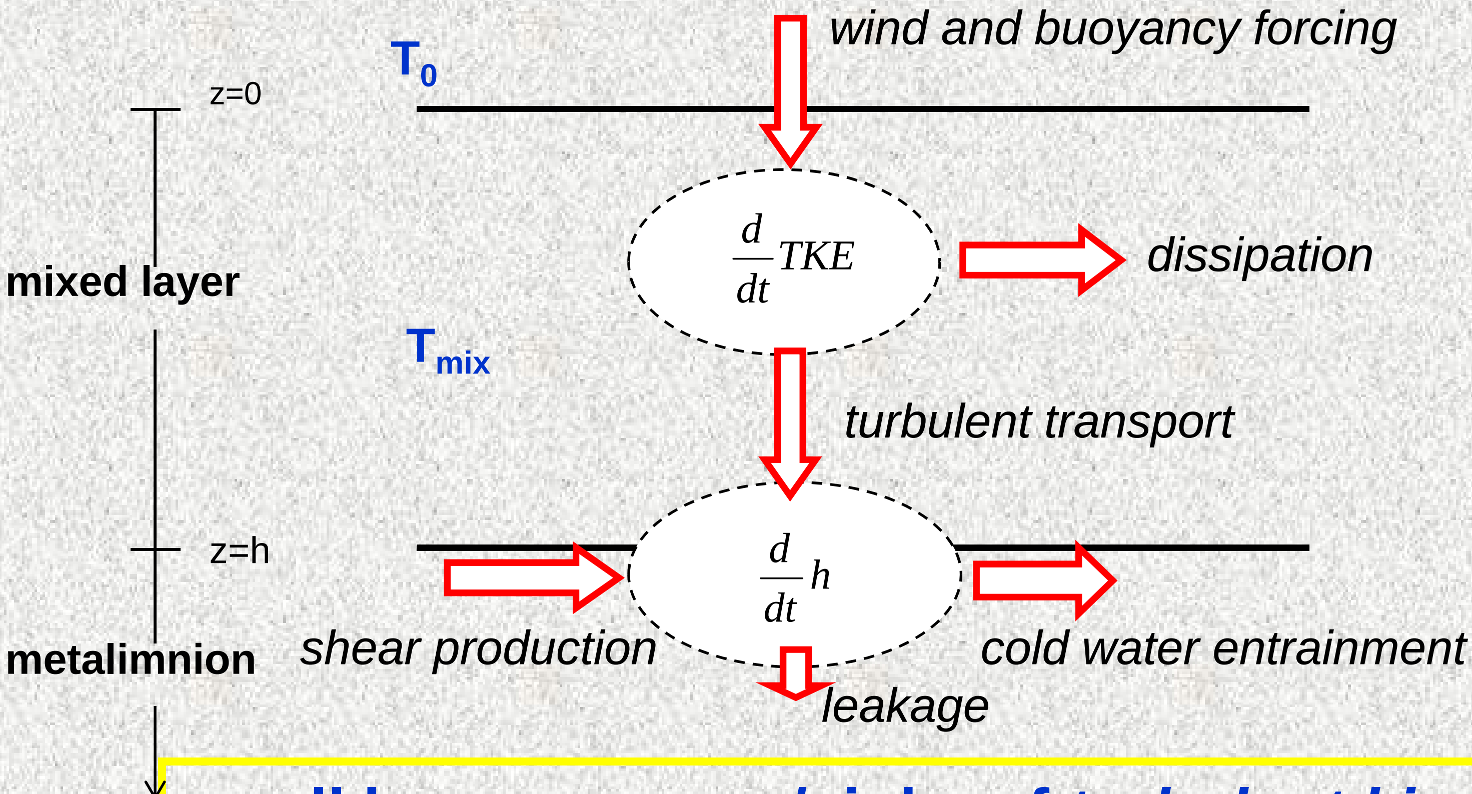


$$F_L = \frac{1}{2} c_L E_s^{3/2}$$

Governing Equations

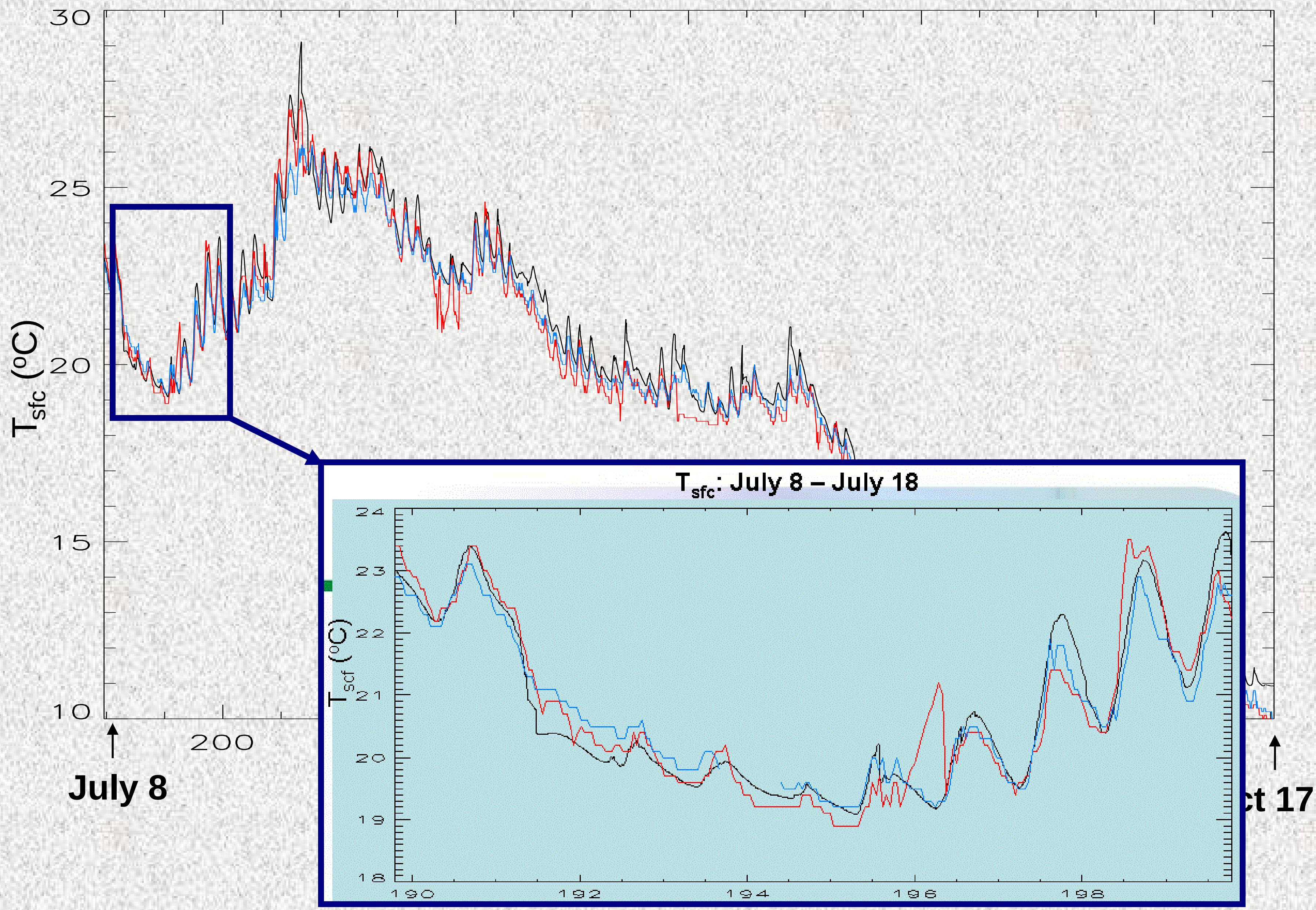
$$\frac{h}{2}\frac{dE_s}{dt} = F_q - F_d - F_i$$

$$\frac{E_s}{2}\frac{dh}{dt} = F_i + F_s - F_p - F_L$$

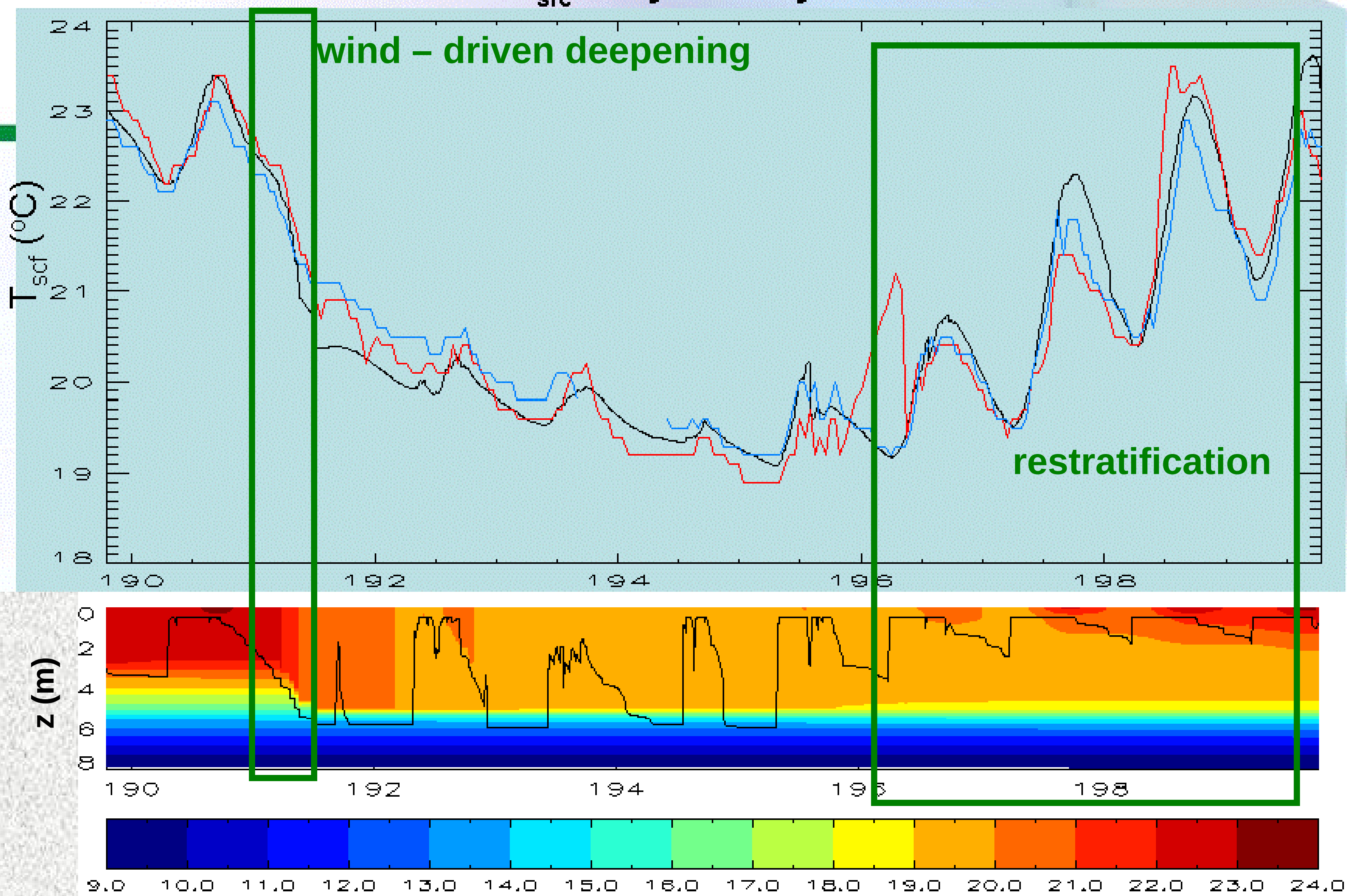


- **well know sources/sinks of *turbulent kinetic energy* govern depth of mixed layer**

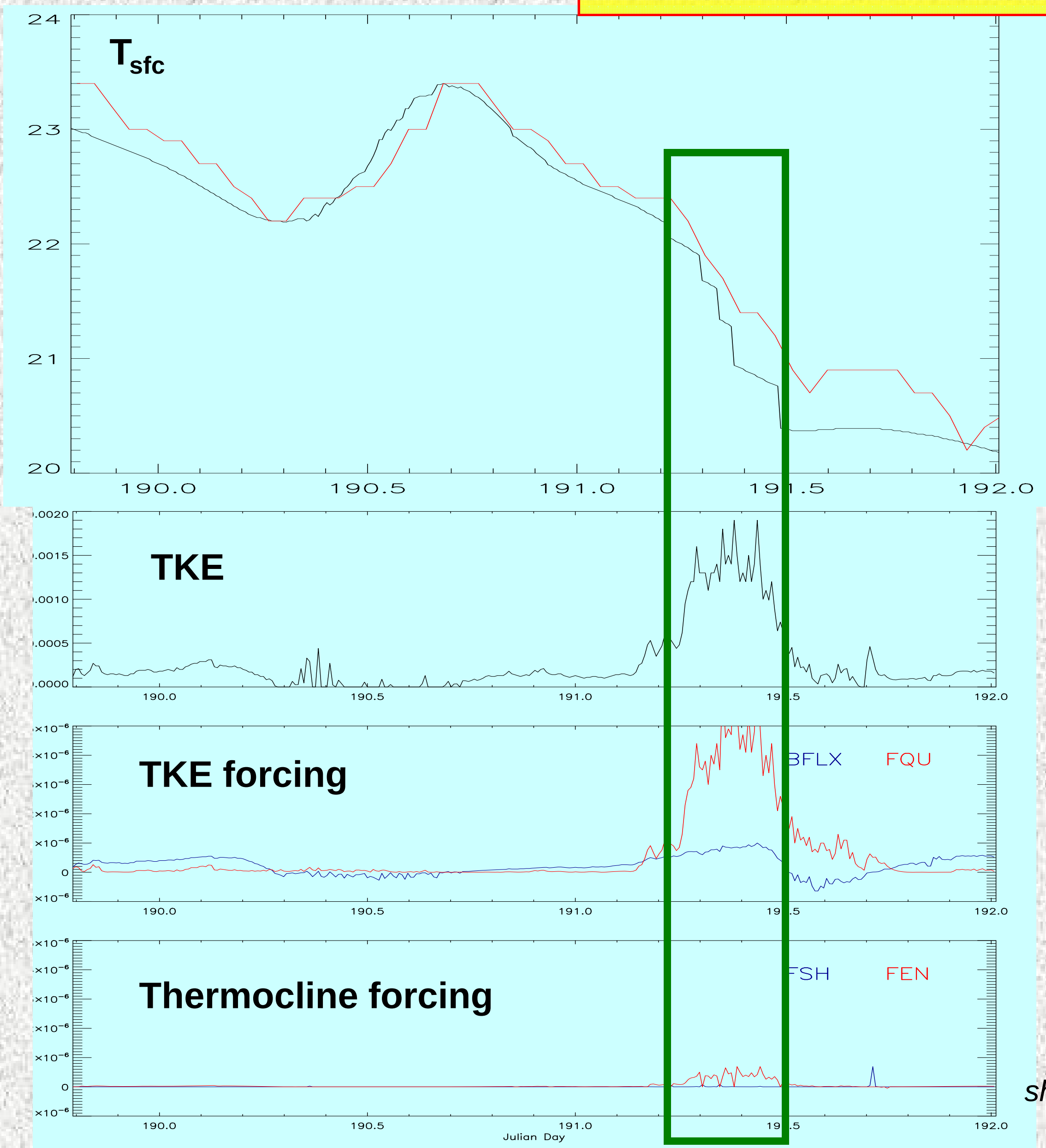
L239 Surface Temperature



T_{sfc}: July 8 – July 18



wind – driven deepening

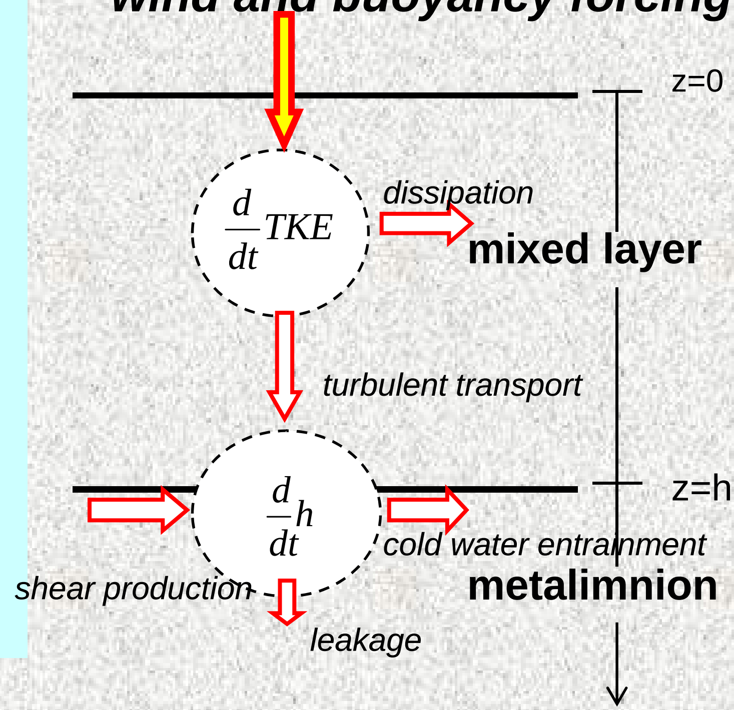


TKE Forcing

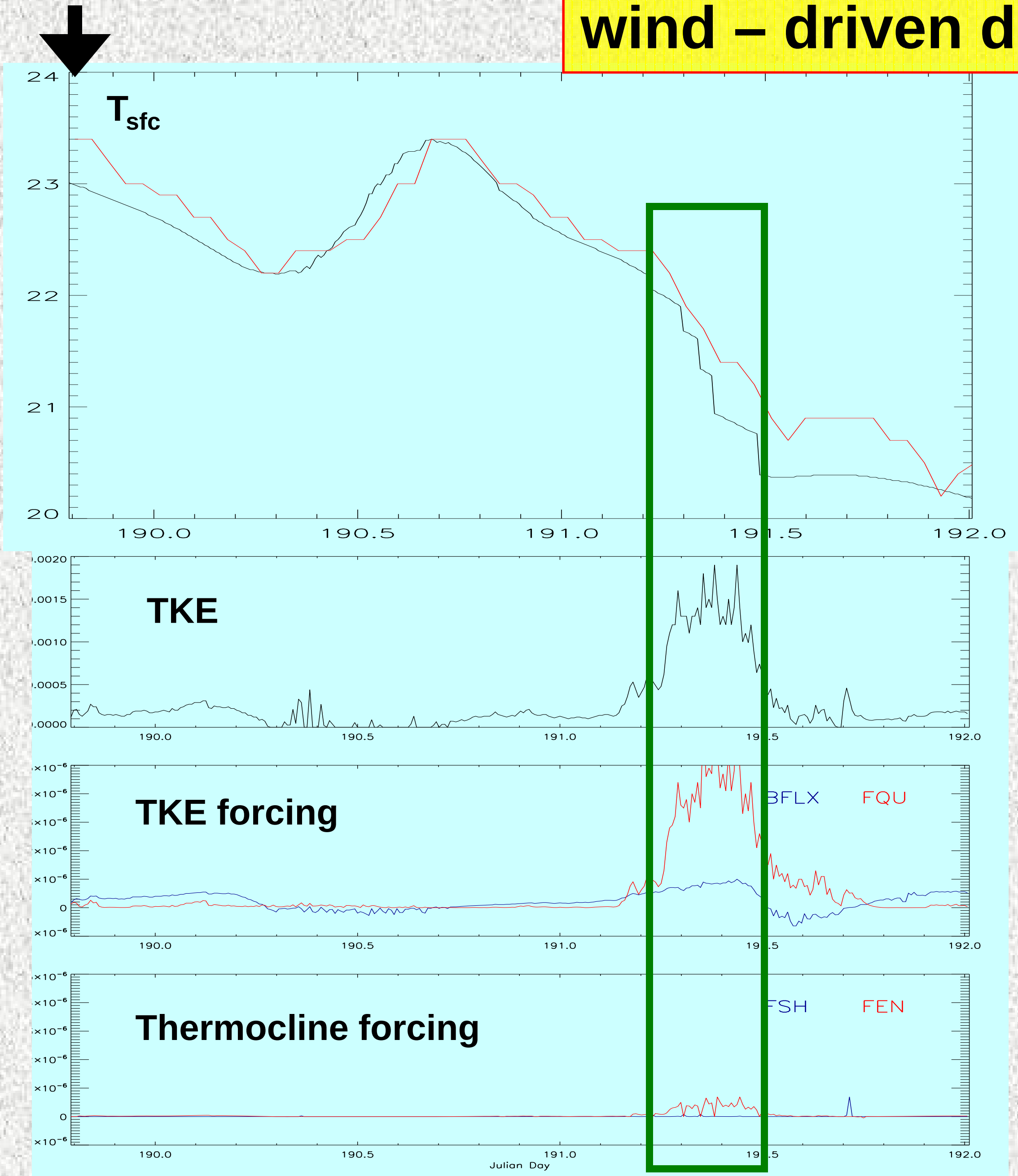
$$F_q = \frac{1}{2}\left(w_*^3 + c_n^3 u_*^3\right)$$

wind stress

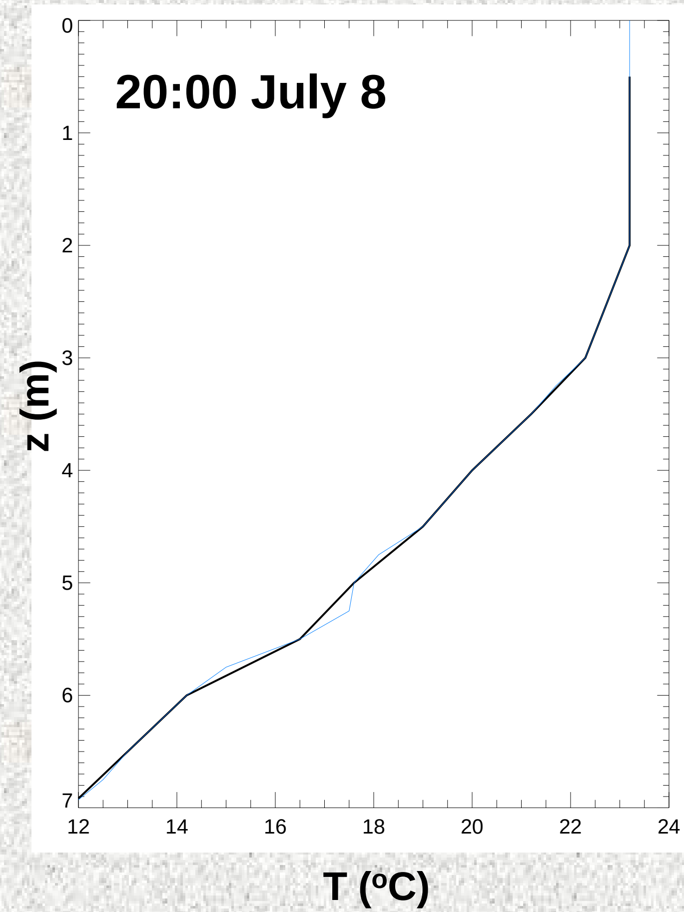
wind and buoyancy forcing



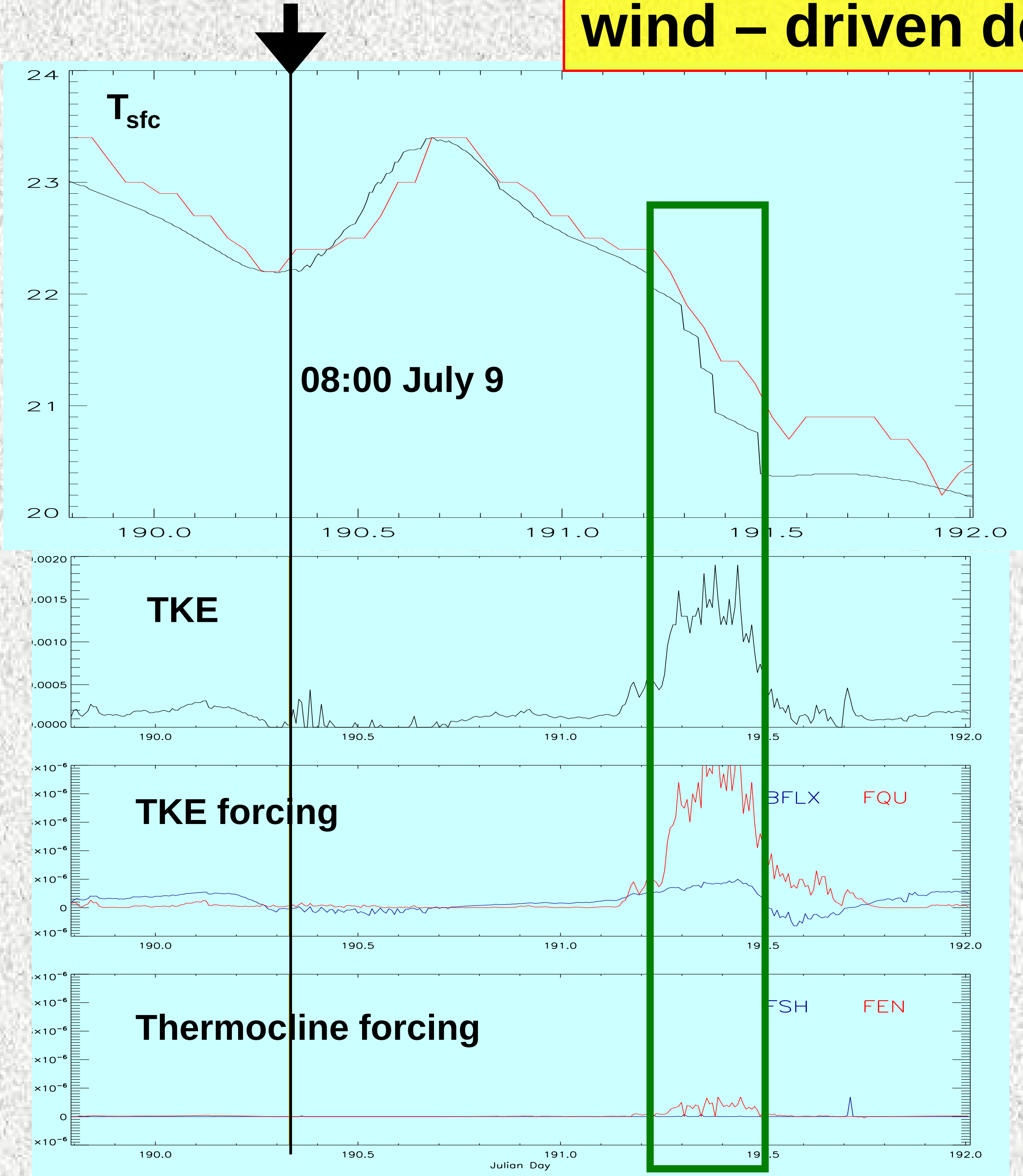
wind – driven deepening

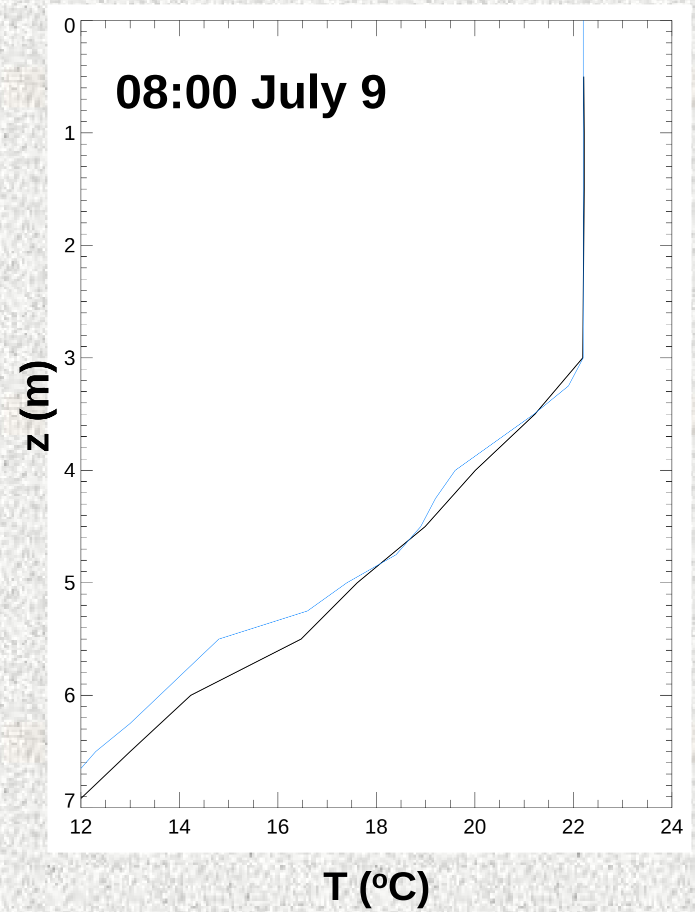


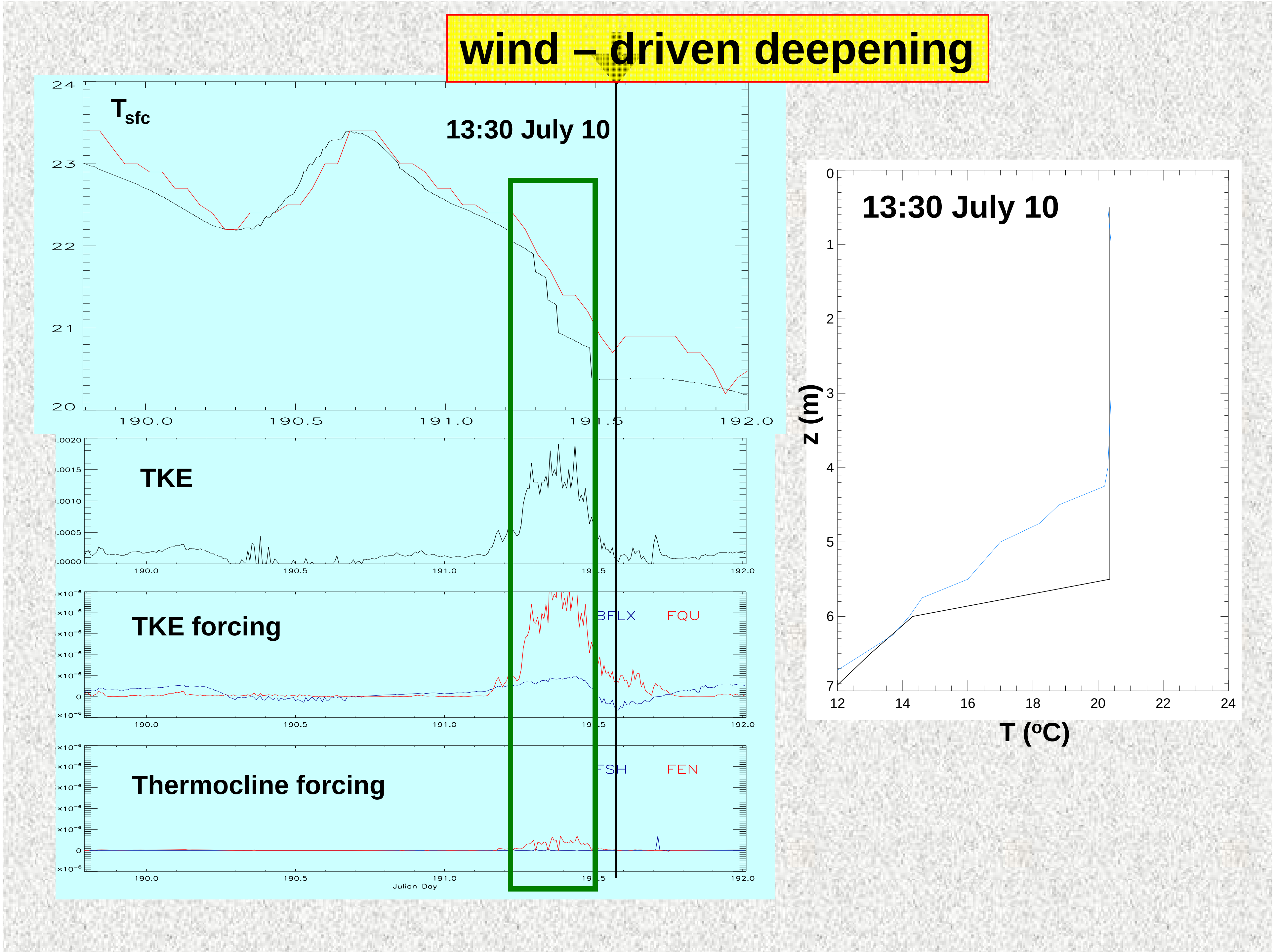
Initial Conditions



wind – driven deepening

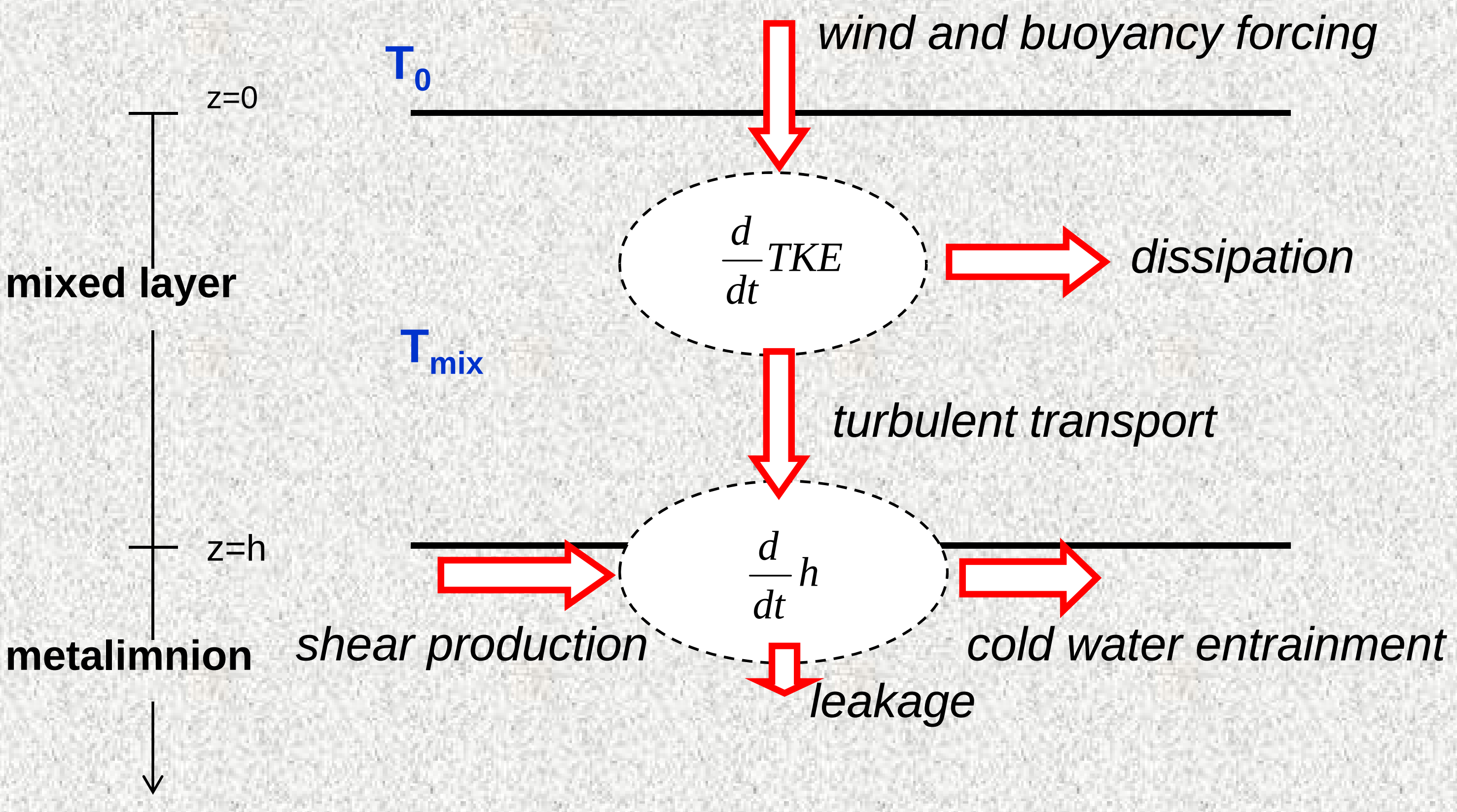






wind – driven deepening
13:30 July 10
T_{sfc}
TKE
TKE forcing
BFLX
FQU
Thermocline forcing
FSH
FEN
Julian Day
13:30 July 10
z (m)
T (°C)

CLASS Lake Module



Henderson – Sellers (1985)

$$K = \frac{\hat{k} u_* z}{P_0} \exp\left(-k^* z\right)\left[1 + 37 Ri^2\right]^{-1}$$

drift decay coefficient

$$k^* = 6.6\sqrt{\sin \vartheta}\, U^{-1.84}$$

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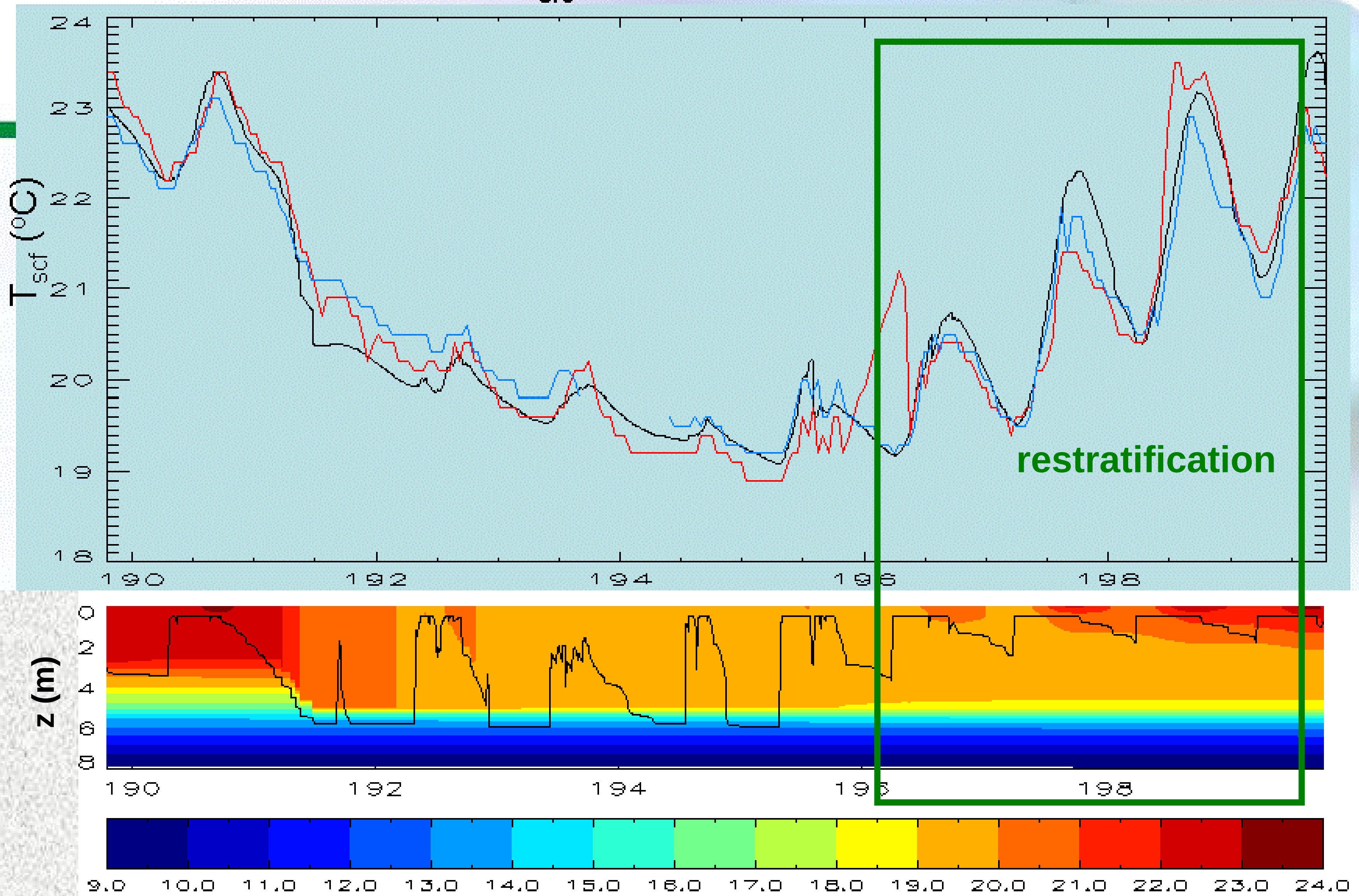
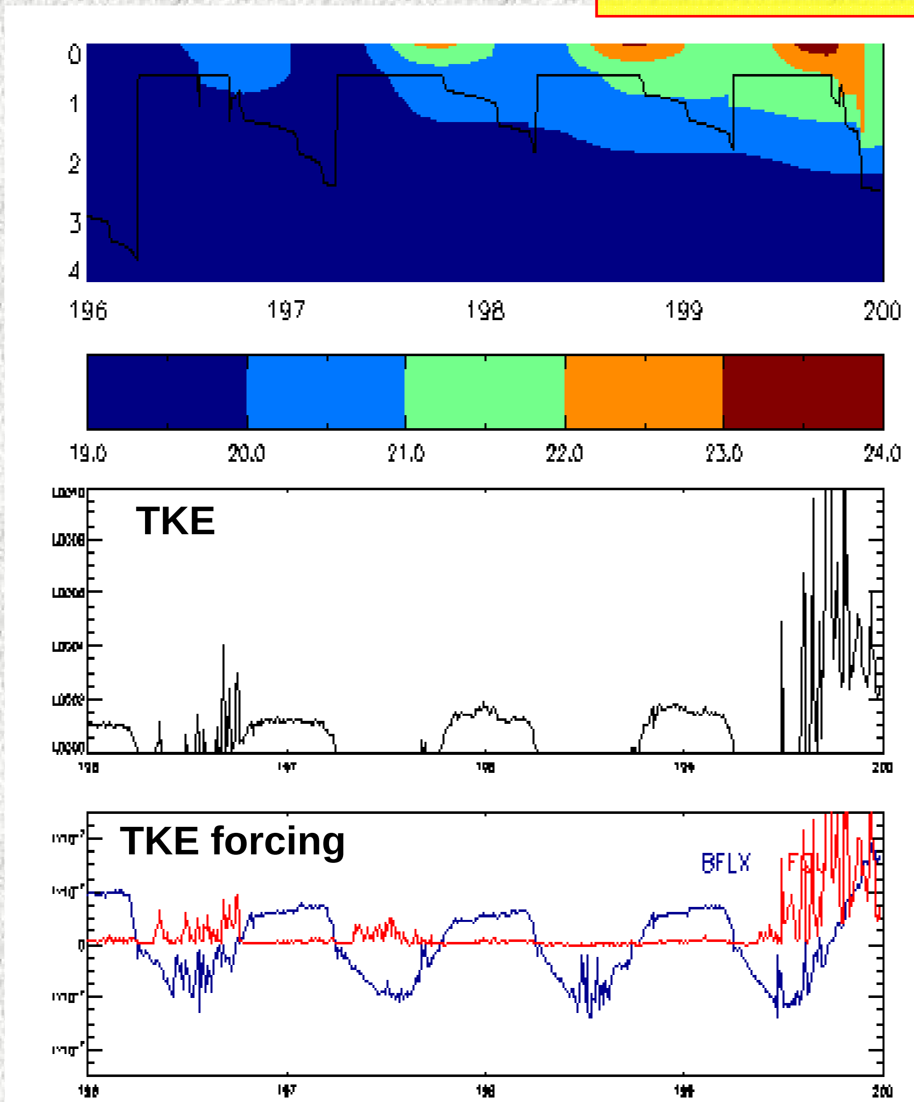


restratification



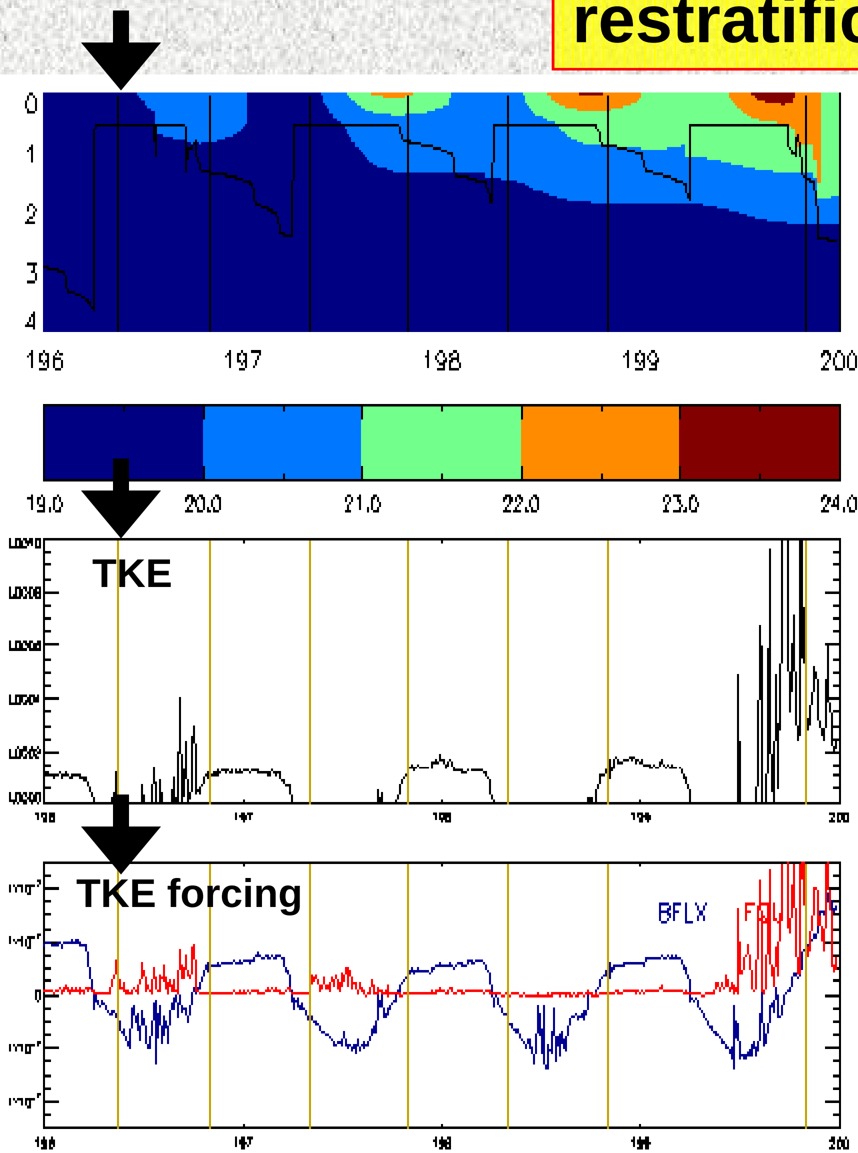
TKE Forcing

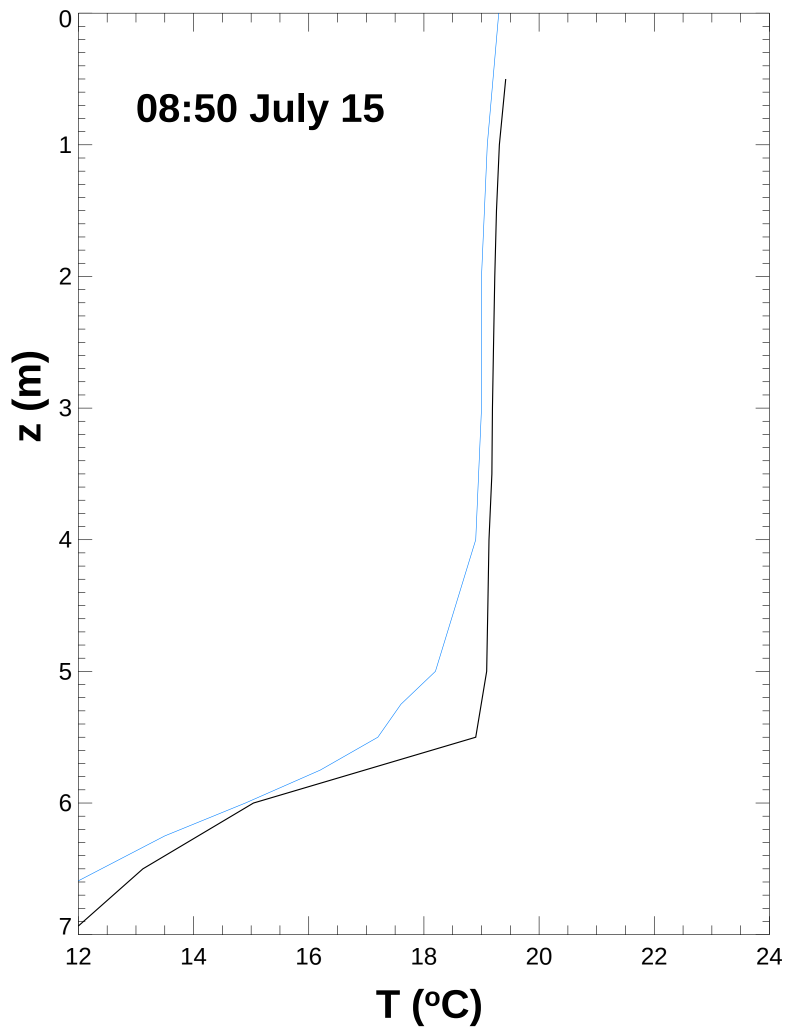
$$F_q = \frac{1}{2}\left(w_*^3 + c_n^3 u_*^3\right)$$

buoyancy

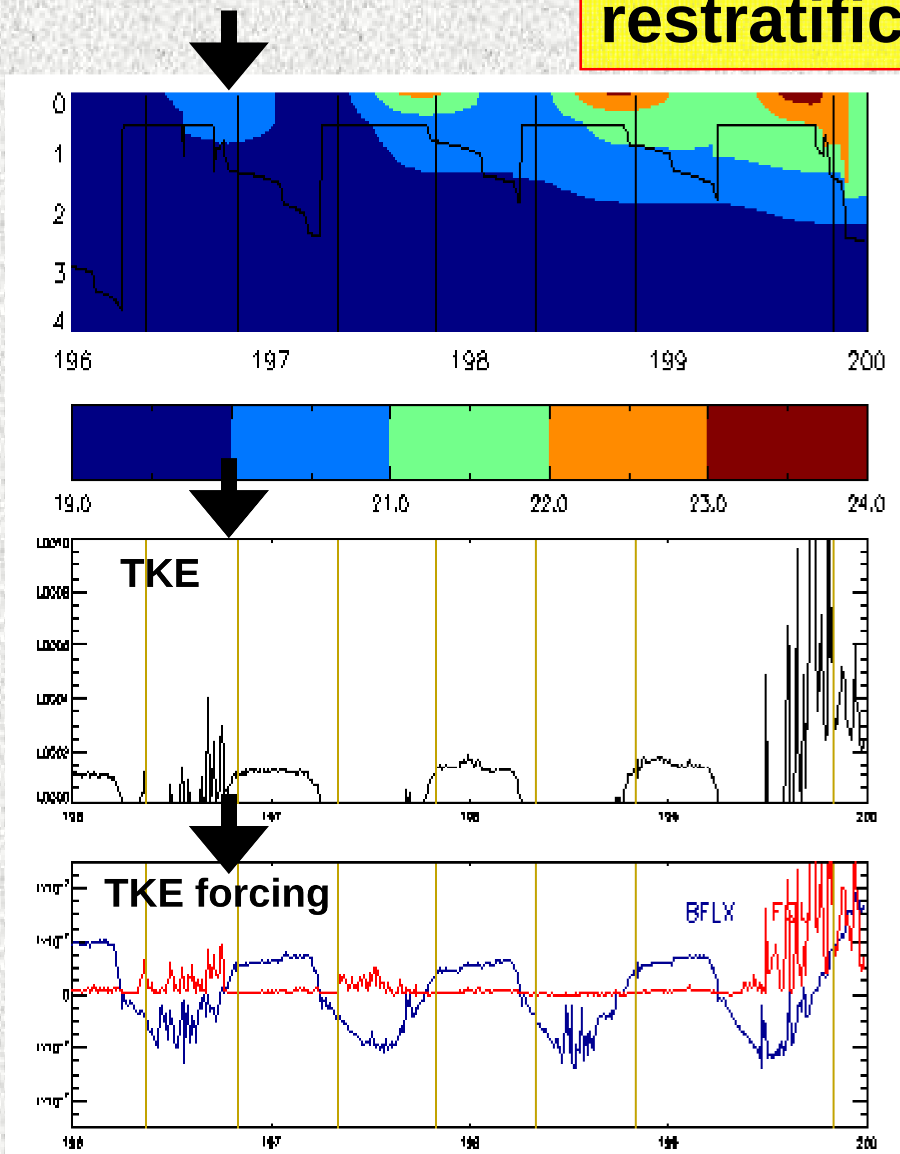
$$-\left[L^* - H_S - H_E\right] - \left[Q^* + Q(h) - \frac{2}{h}\int_0^h Q\,dz\right]$$

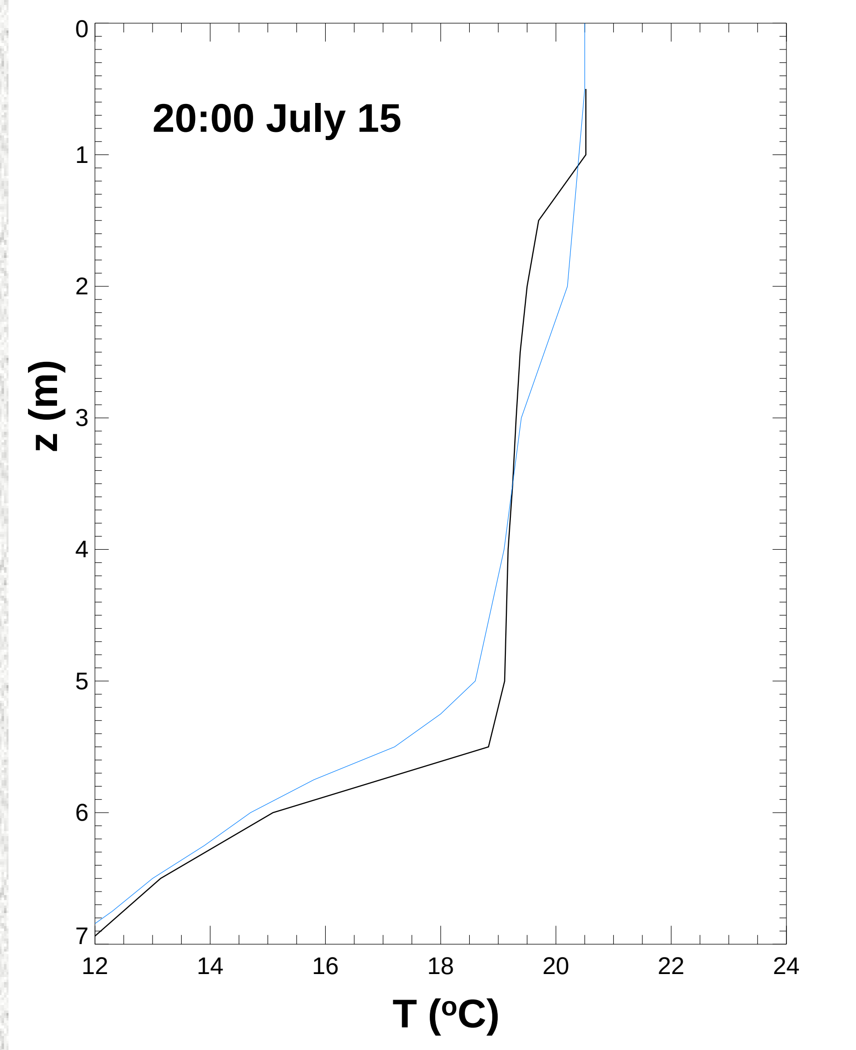
restratification



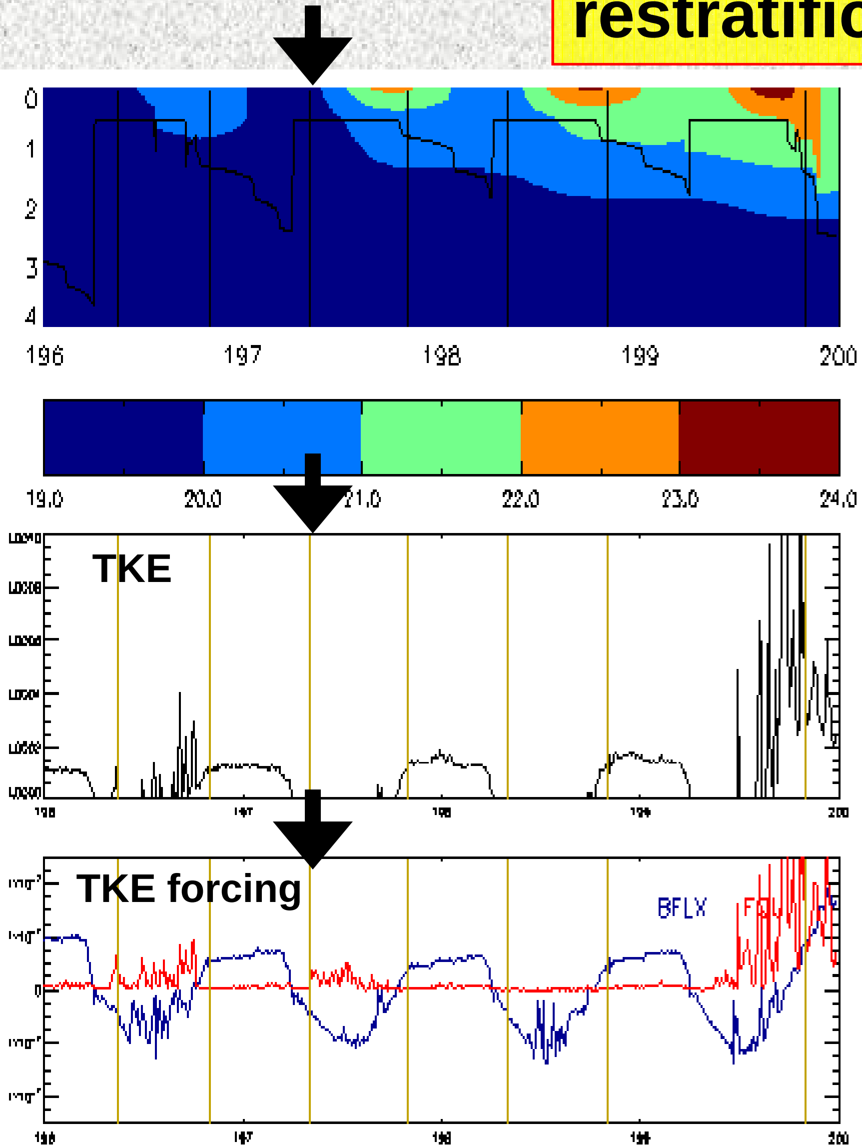


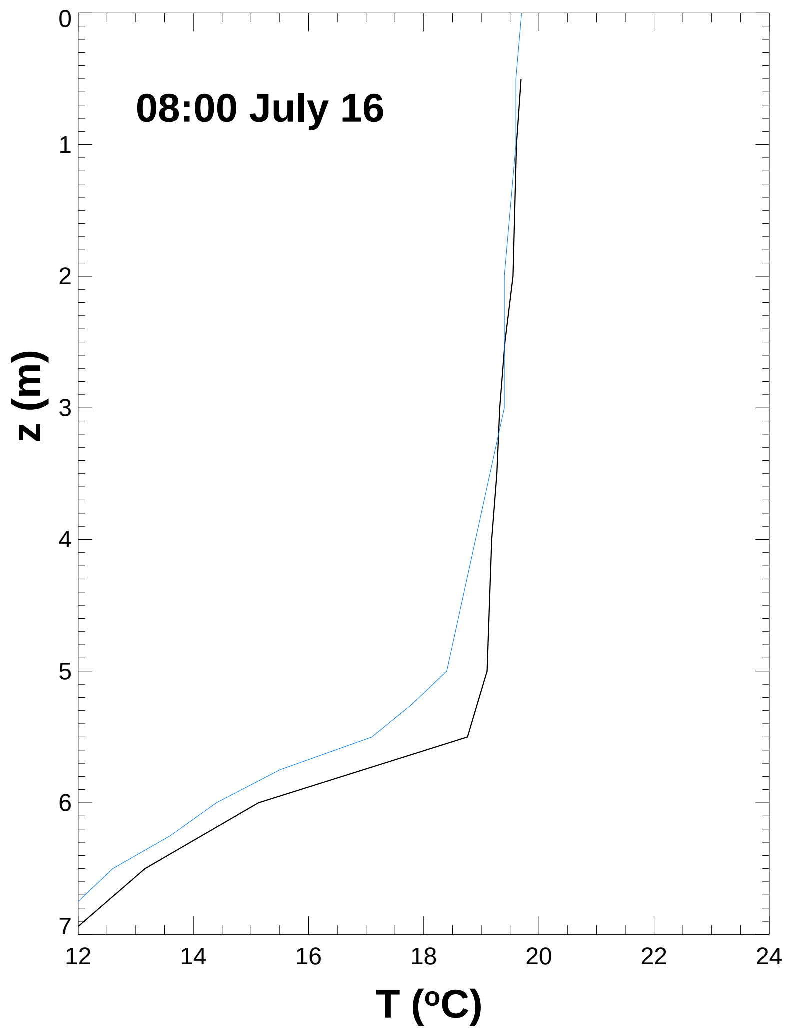
restratification



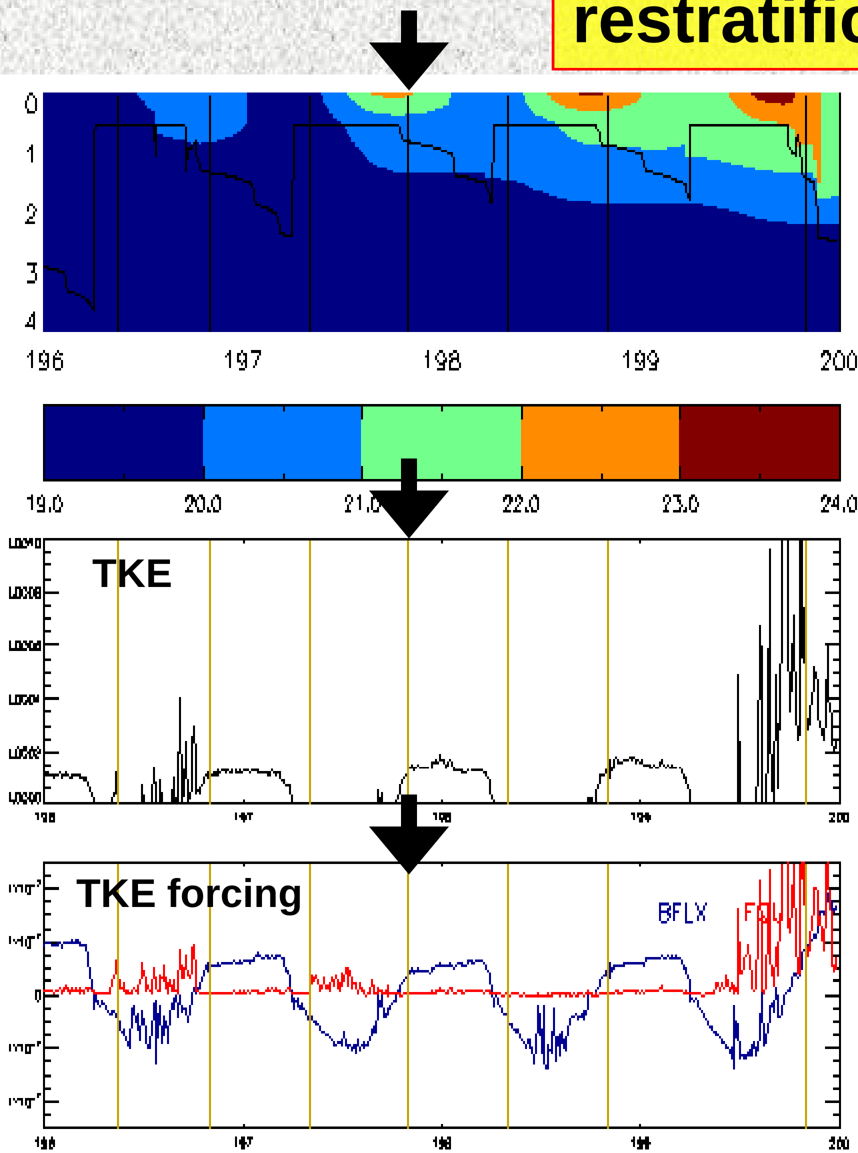


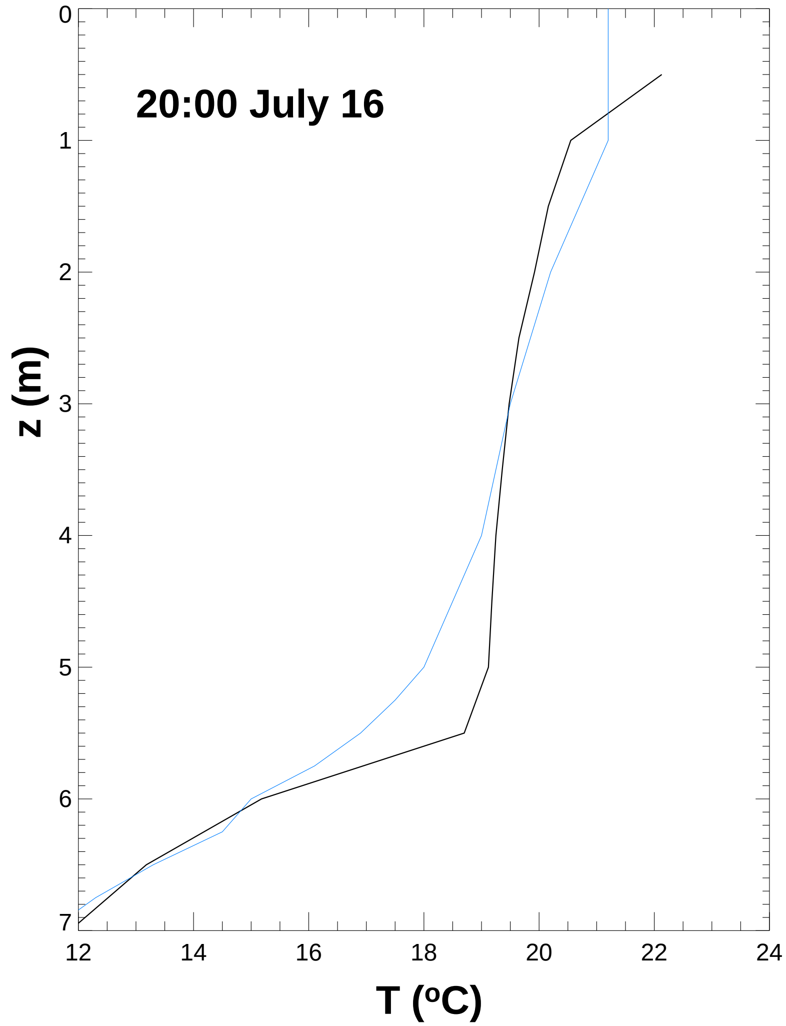
restratification

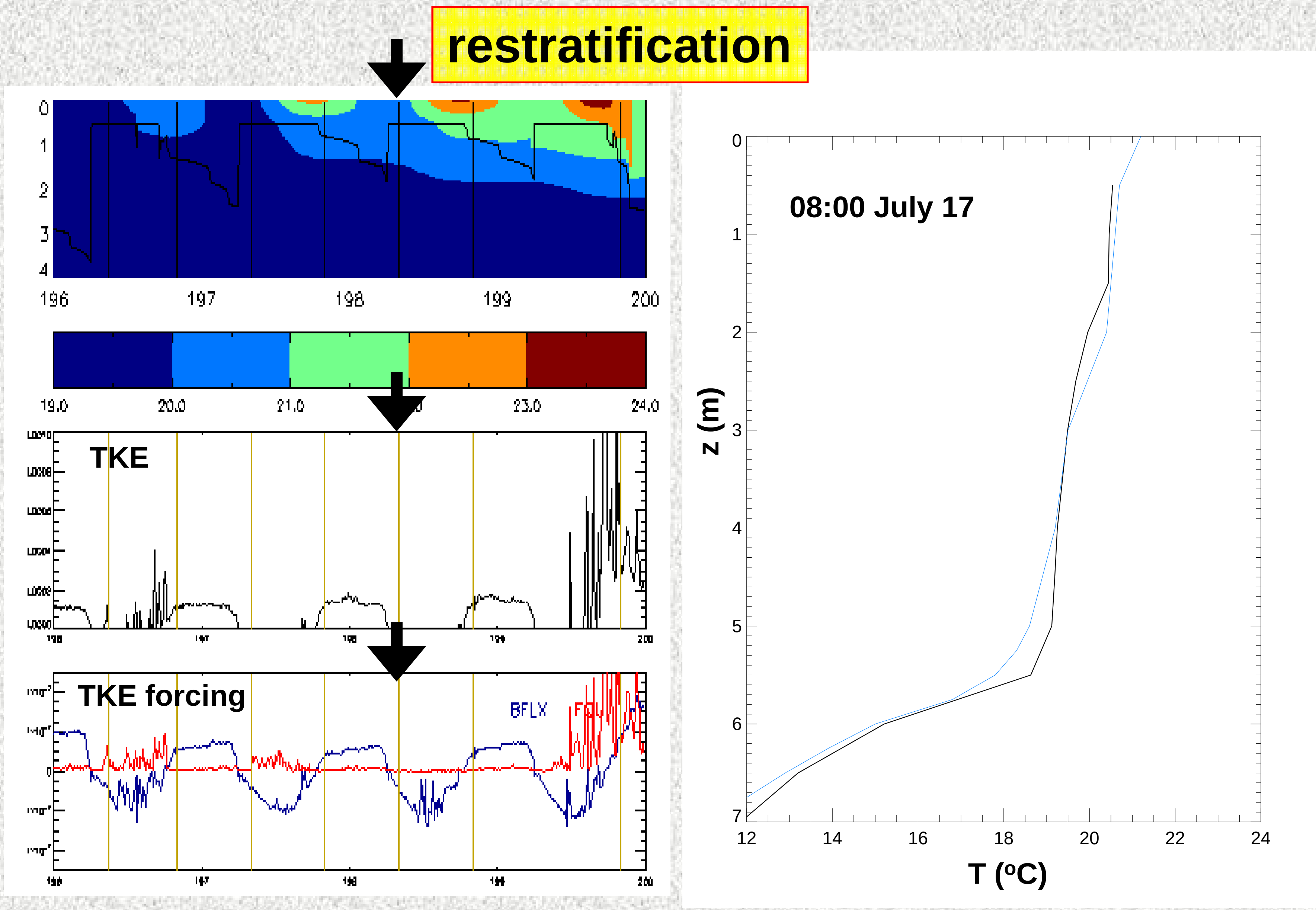




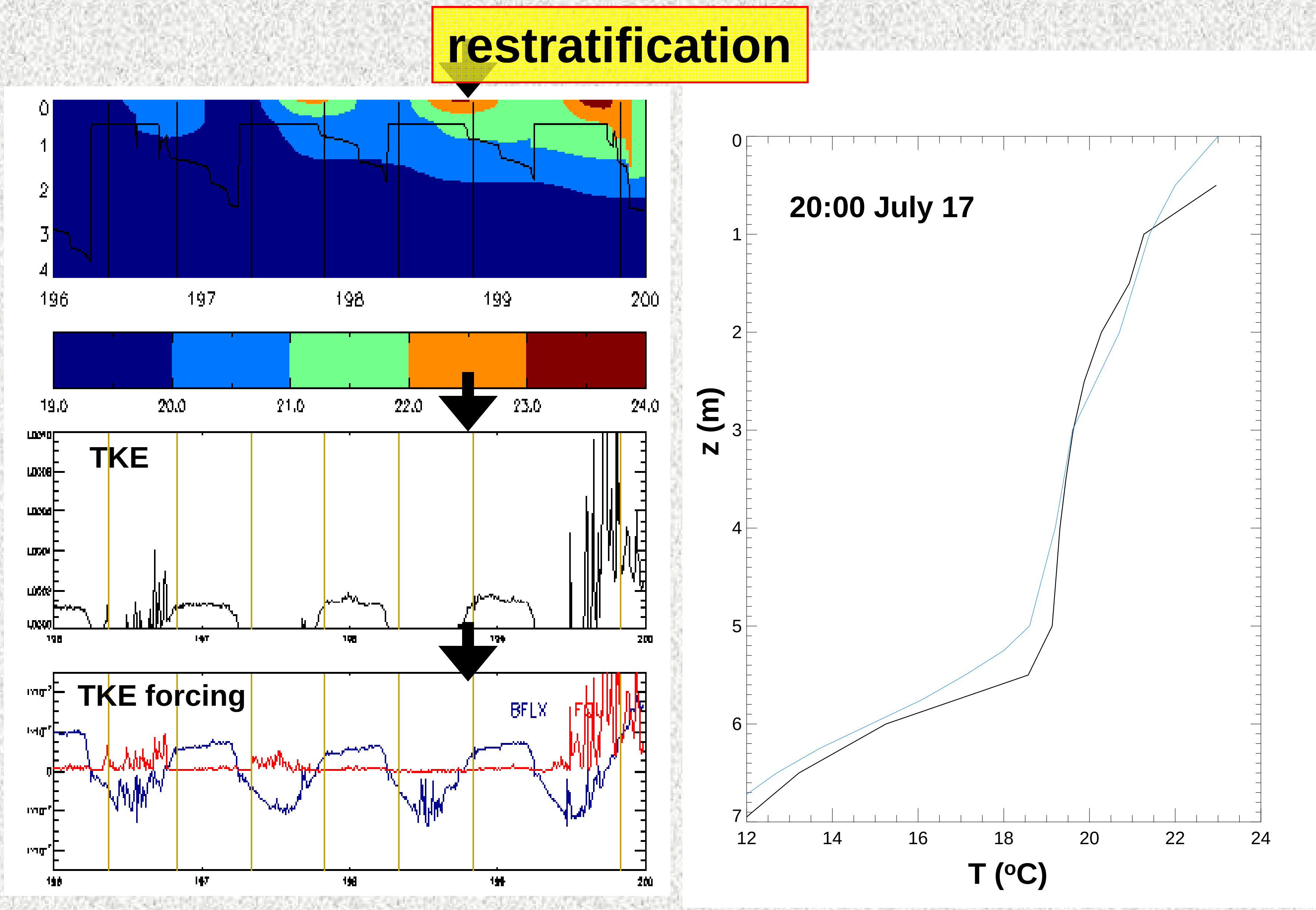
restratification



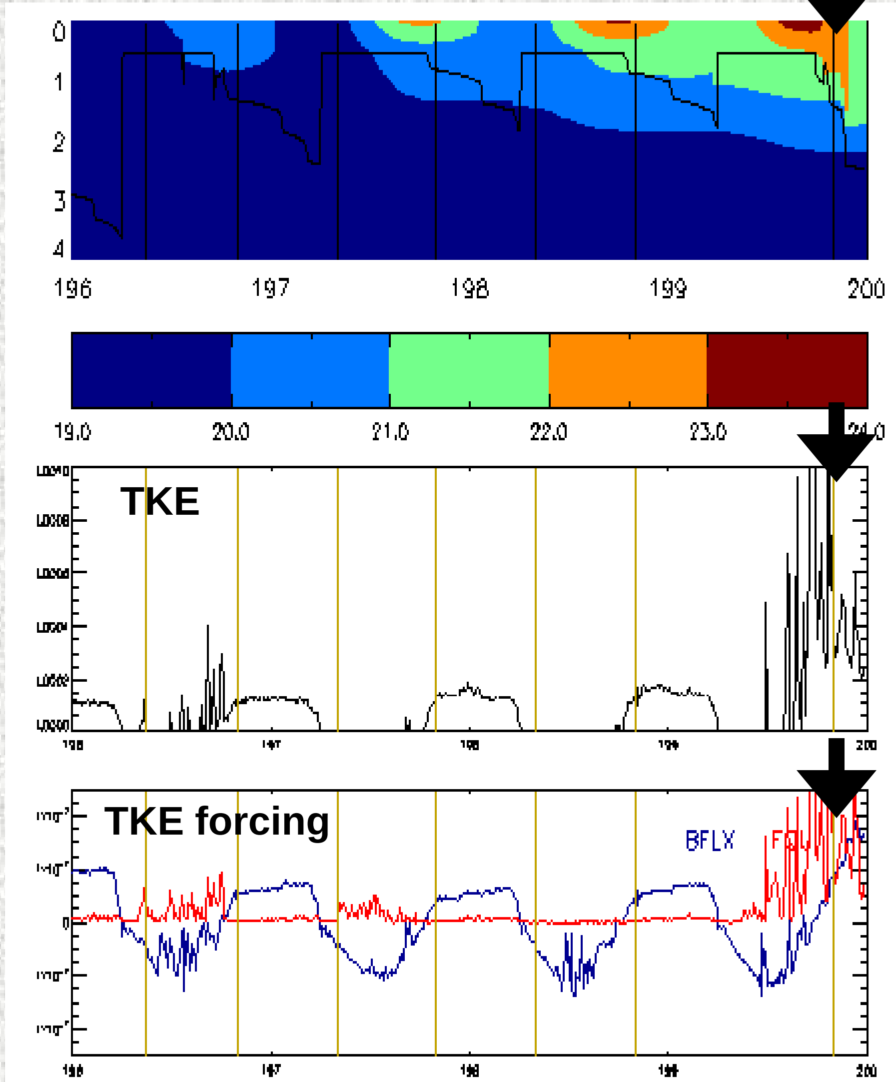


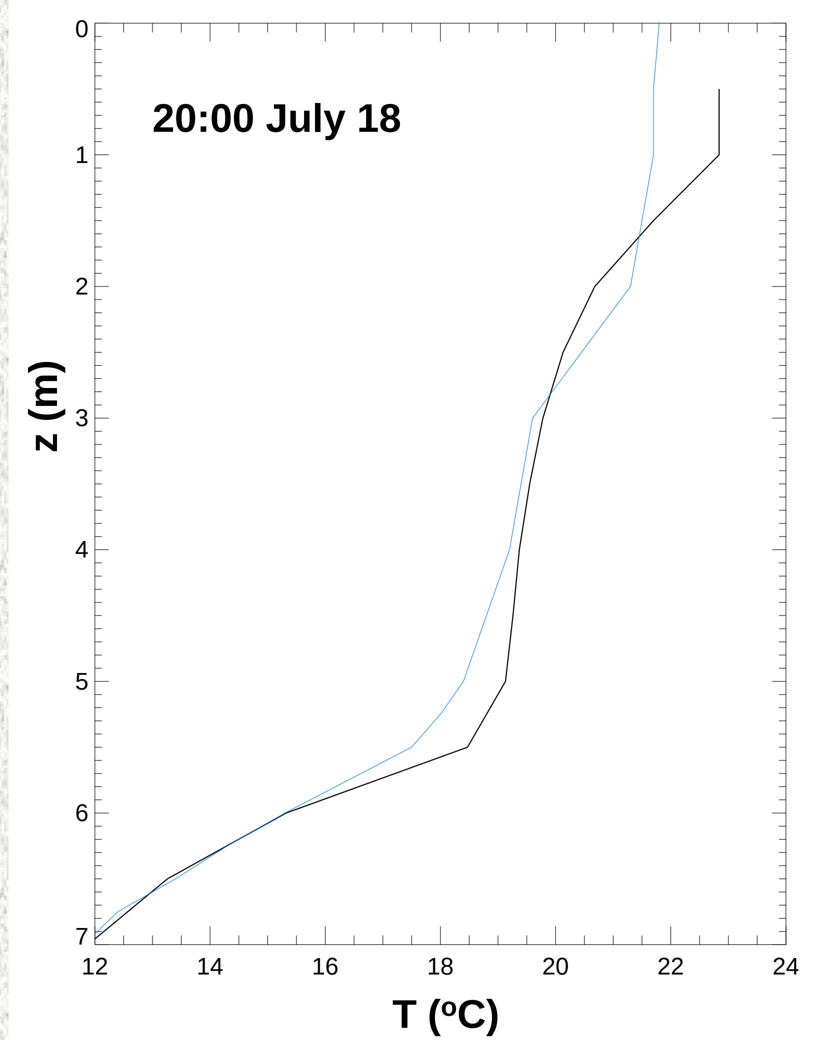


restratification
TKE
TKE forcing
BFLX
08:00 July 17
z (m)
T (°C)
196 197 198 199 200
19.0 20.0 21.0 23.0 24.0
12 14 16 18 20 22 24

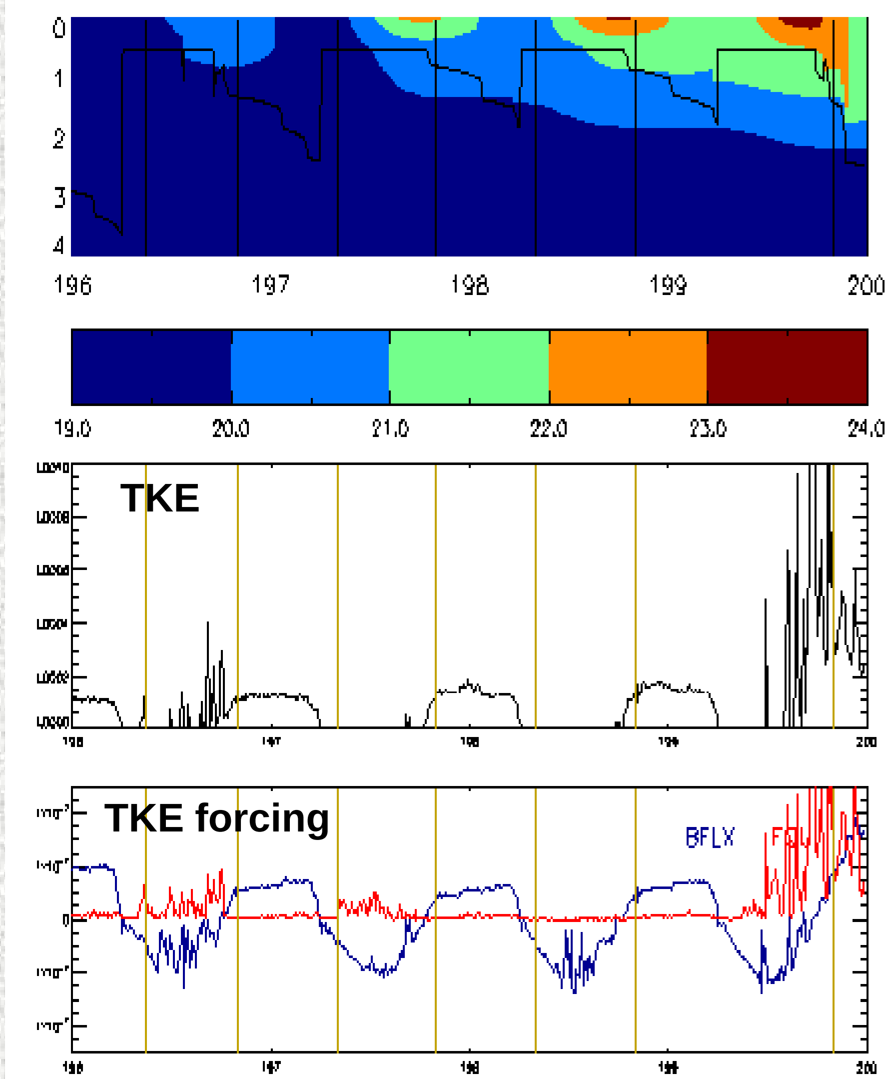

restratification
TKE
TKE forcing
BFLX
20:00 July 17
z (m)
T (°C)

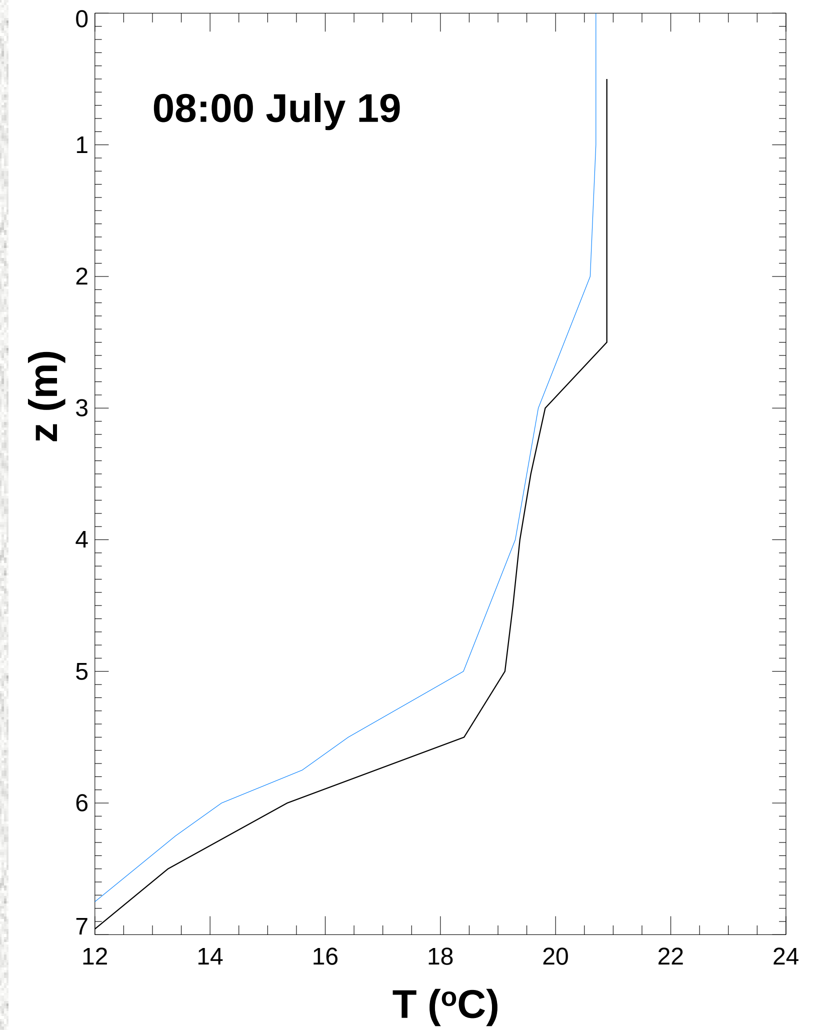
restratification



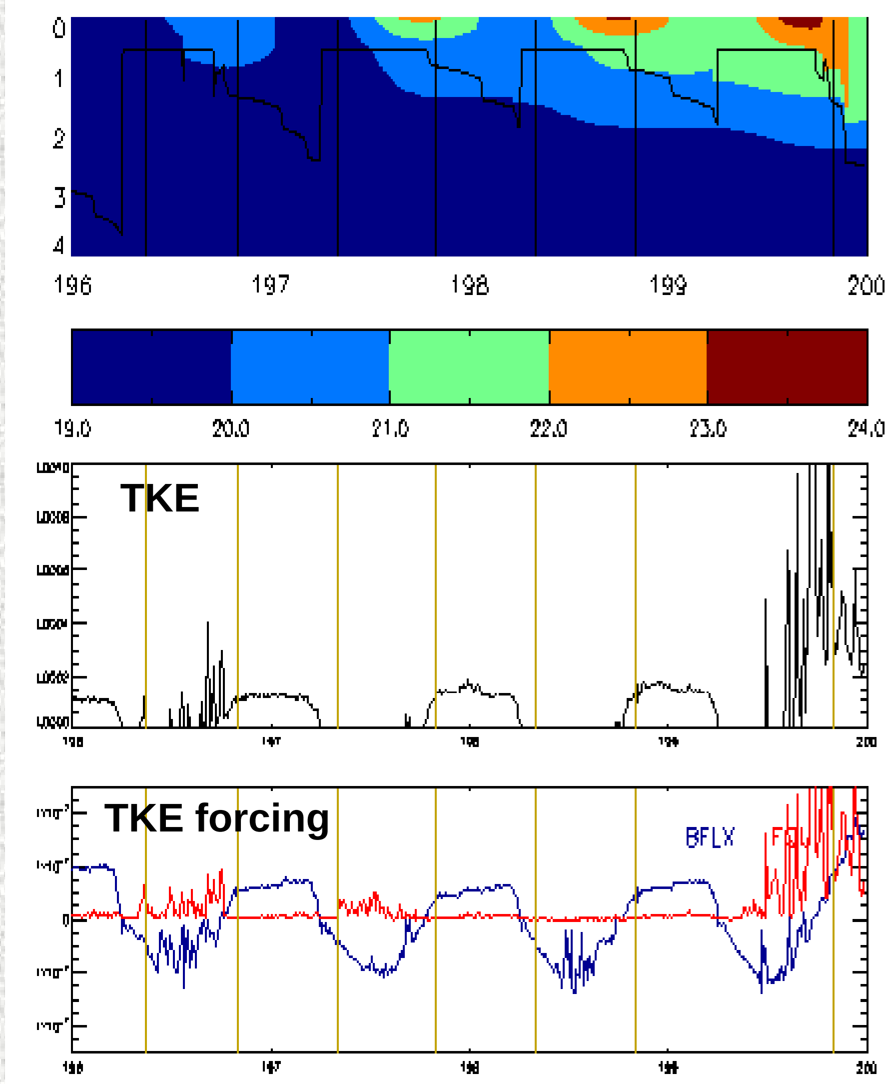


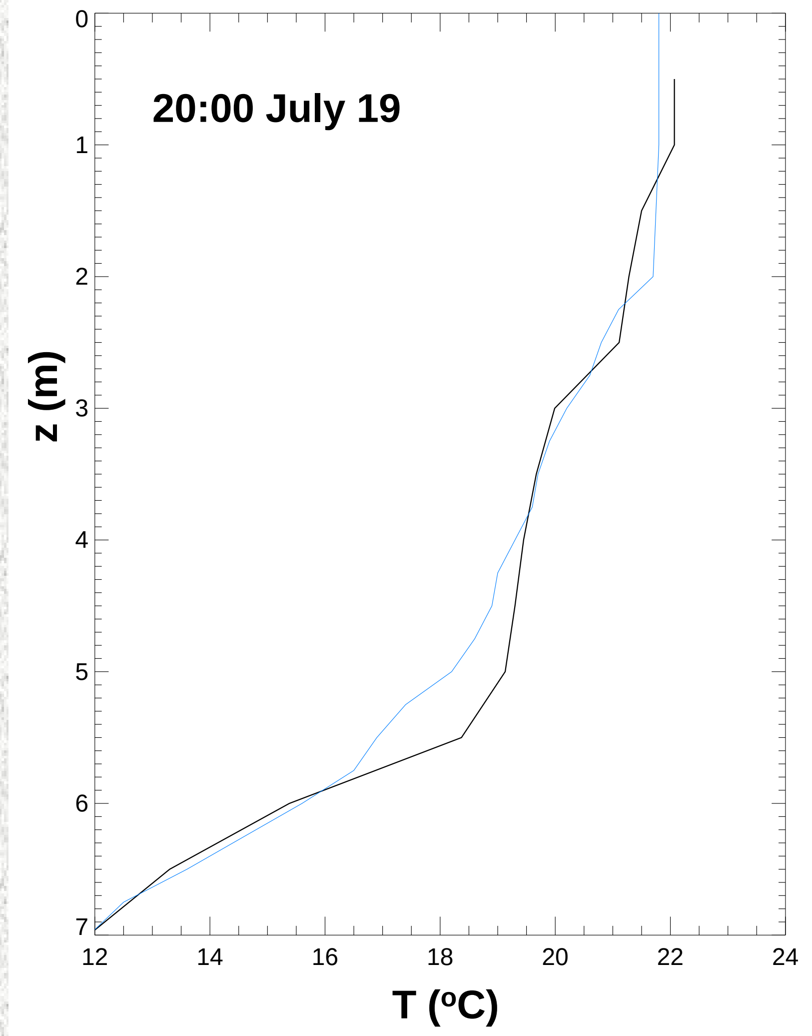
restratification





restratification





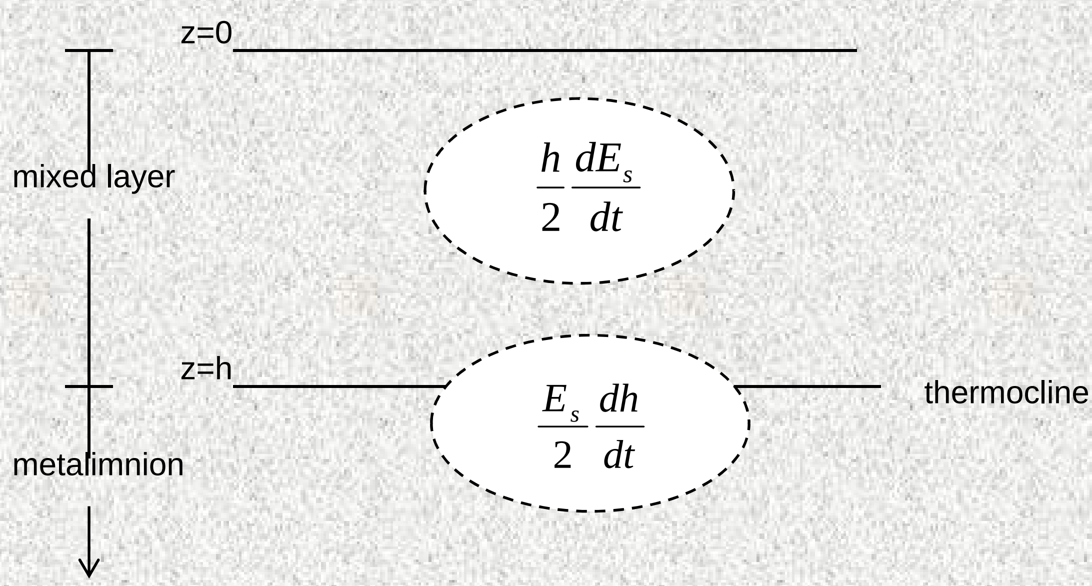
Conclusions

• **Where possible, begin from fundamental law (*e.g. conservation of energy, mass, etc.).***

• **Clearly state (and understand) the assumptions you need to simplify the problem (*e.g. turbulent eddies small for eddy diffusivity concept).***

• **It's better to parameterize processes that cause an effect than it is to parameterize the effect itself (*e.g. rather than parameterize a temperature profile, it's better to parameterize processes that move heat)***

LAKE1D – Turbulence Model



$$\frac{d}{dt}\left(\frac{1}{2}hE_s\right)=\frac{h}{2}\frac{dE_s}{dt}+\frac{E_s}{2}\frac{dh}{dt}$$

$$=F_q-F_d+F_s-F_p$$