Micro-scale to Meso-scale: An update on the IP3 sub-grid soil-water budget

by

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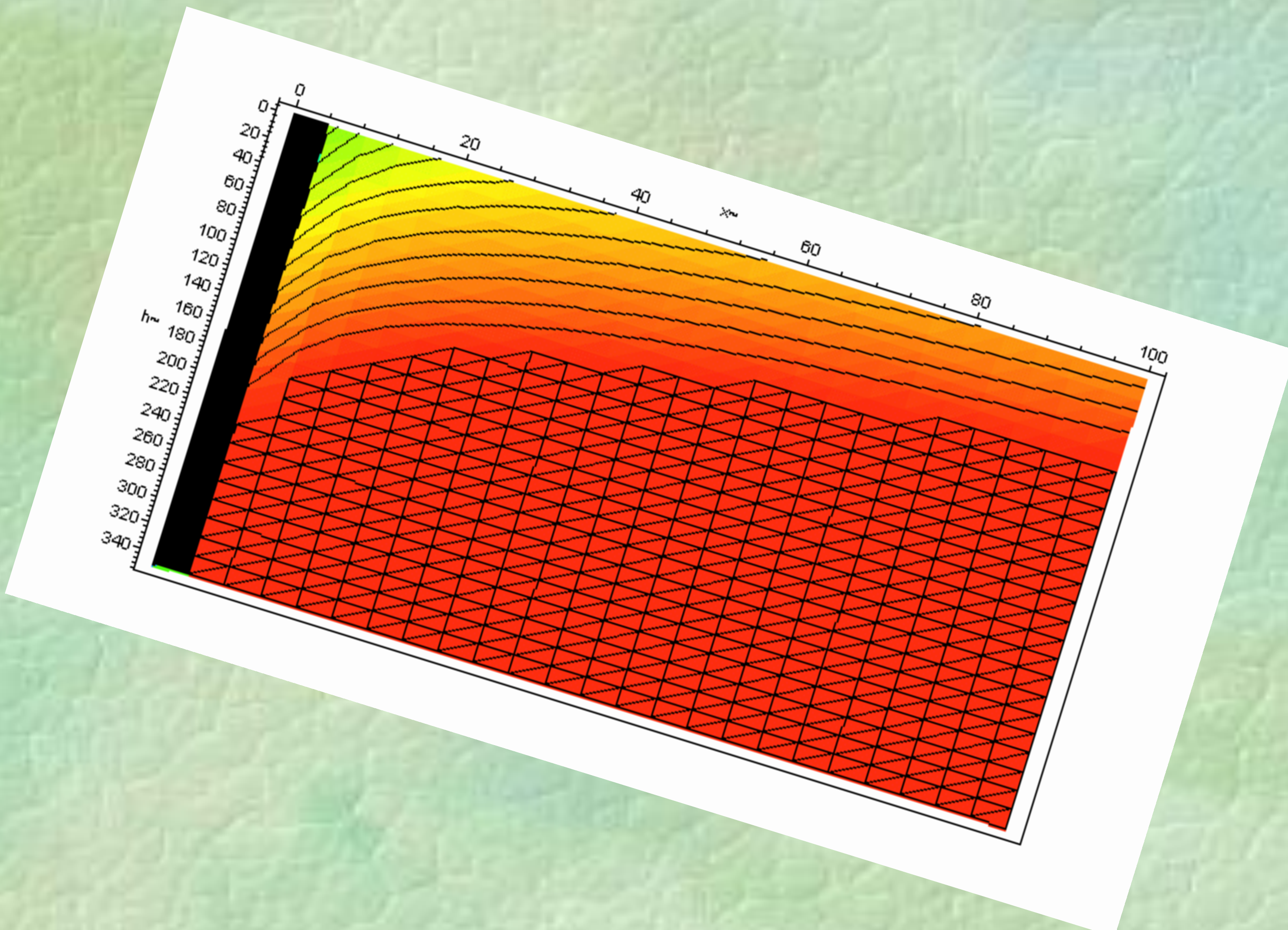
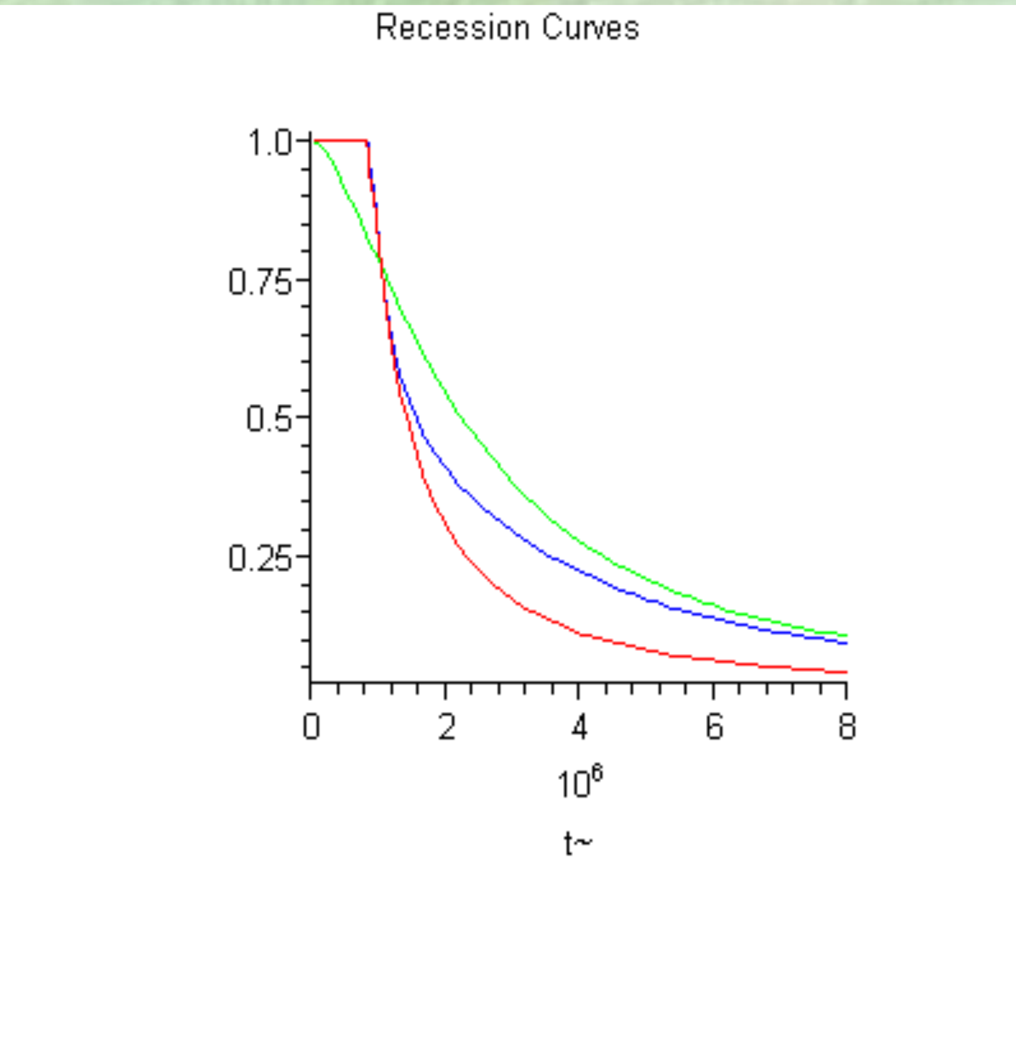
- **<u>OUTLINE</u>**

- Parameterization of soil water must be simple and robust.
- A simple soil moisture parameterization scheme, WATDRAIN, was previously developed for shallow aquifers.
- A new scheme presented, is based on more rigorous application of Richard’s Equation.

MAGS Tile

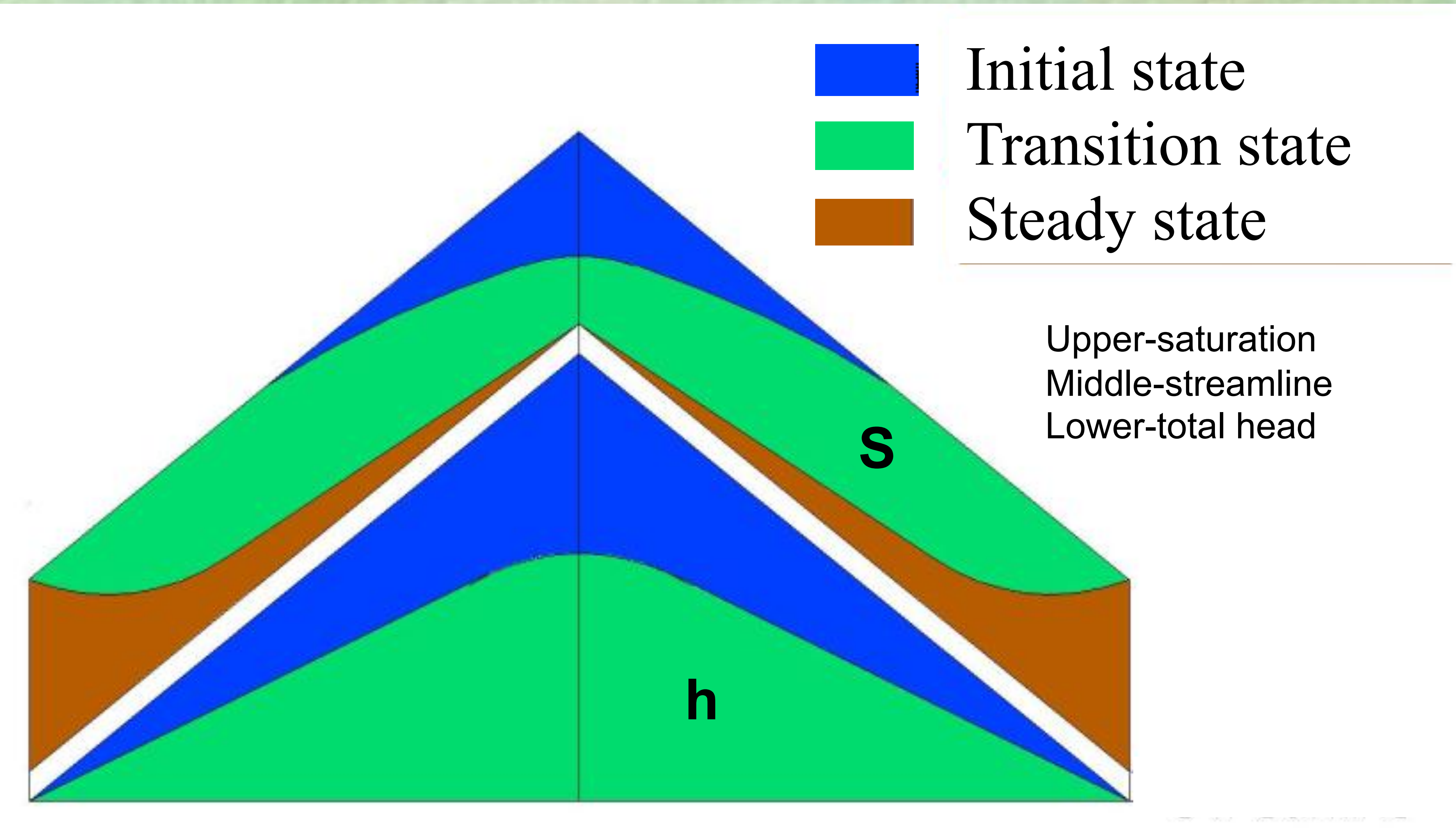


0
20
40
60
80
100
x~
h~
0
20
40
60
80
100
120
140
160
180
200
220
240
260
280
300
320
340

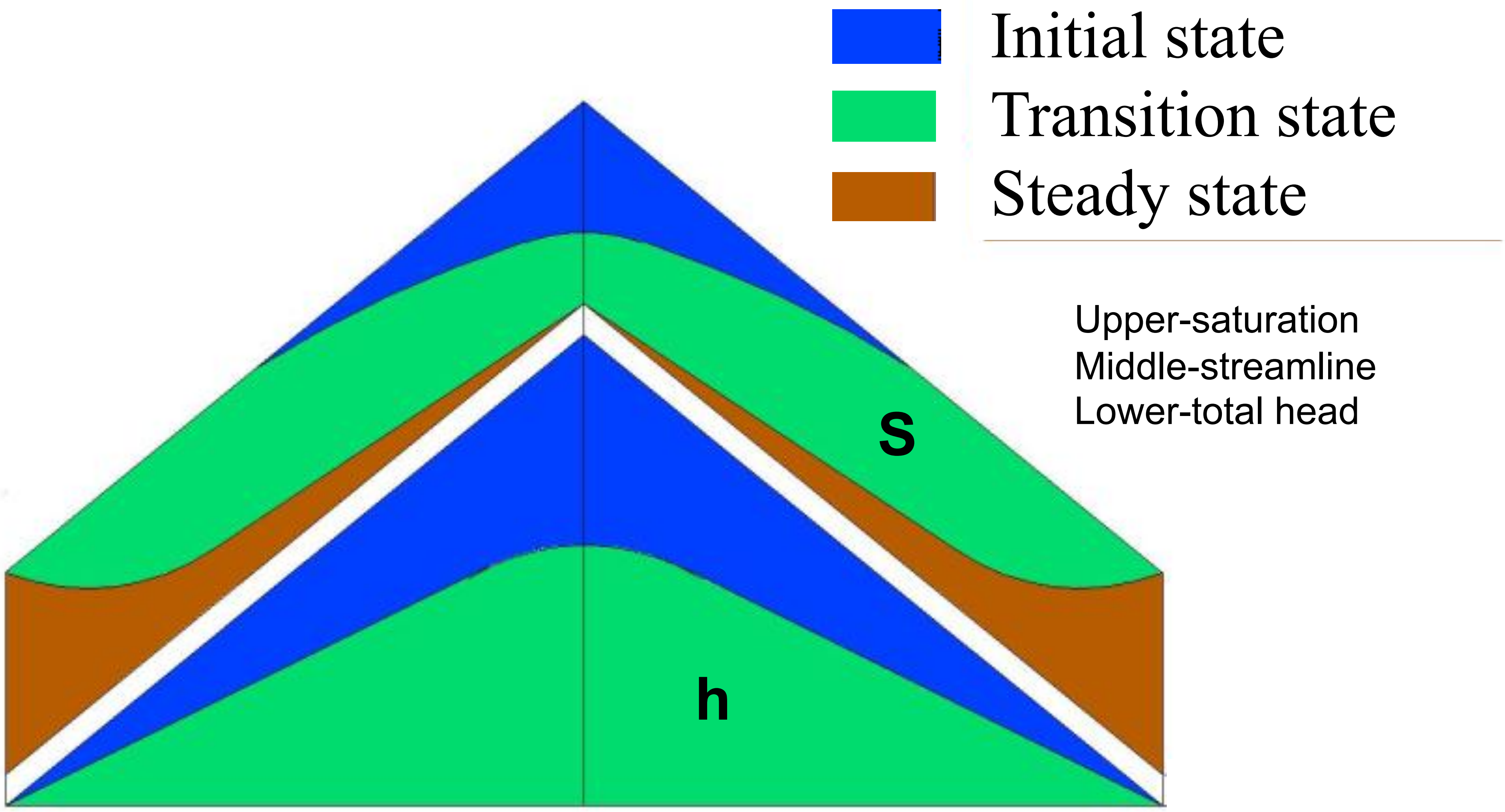


Red line is a typical gravity dominated curve. Green line is the corresponding suction dominated solution. WATDrainV2 uses an empirical blend of these. WATDrainV3 will use Equation (1) which is the blue line in Figure 3.

IP3 Tile?


Initial state
Transition state
Steady state
Upper-saturation
Middle-streamline
Lower-total head
s
h

IP3 Tile?



Upper-saturation
Middle-streamline
Lower-total head

- **METHODOLOGY**
- A shallow aquifer with length *L* and slope Delta *Δ* is used to develop the solution (Figure 1). The seepage face is always saturated and the upper one has a no flow boundary condition (B.C.) .

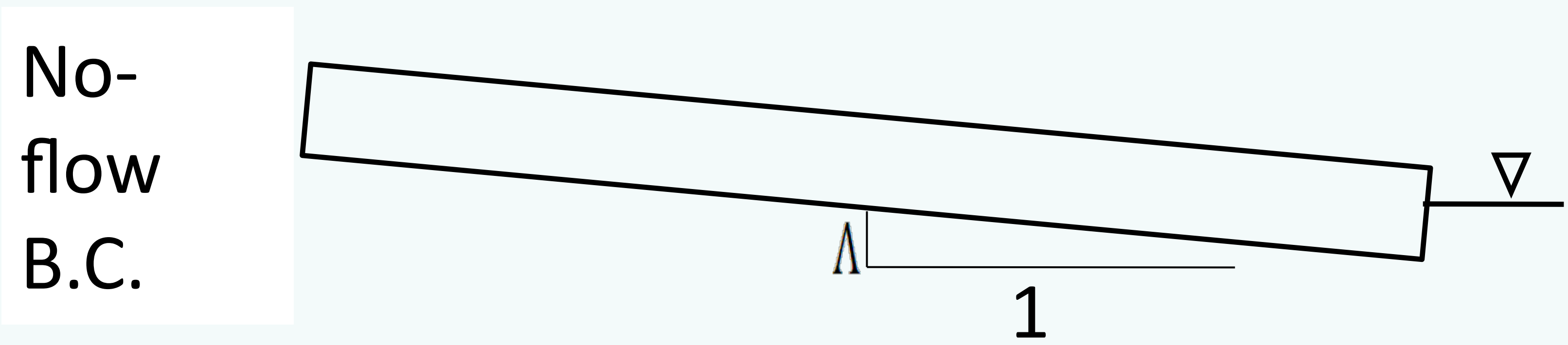
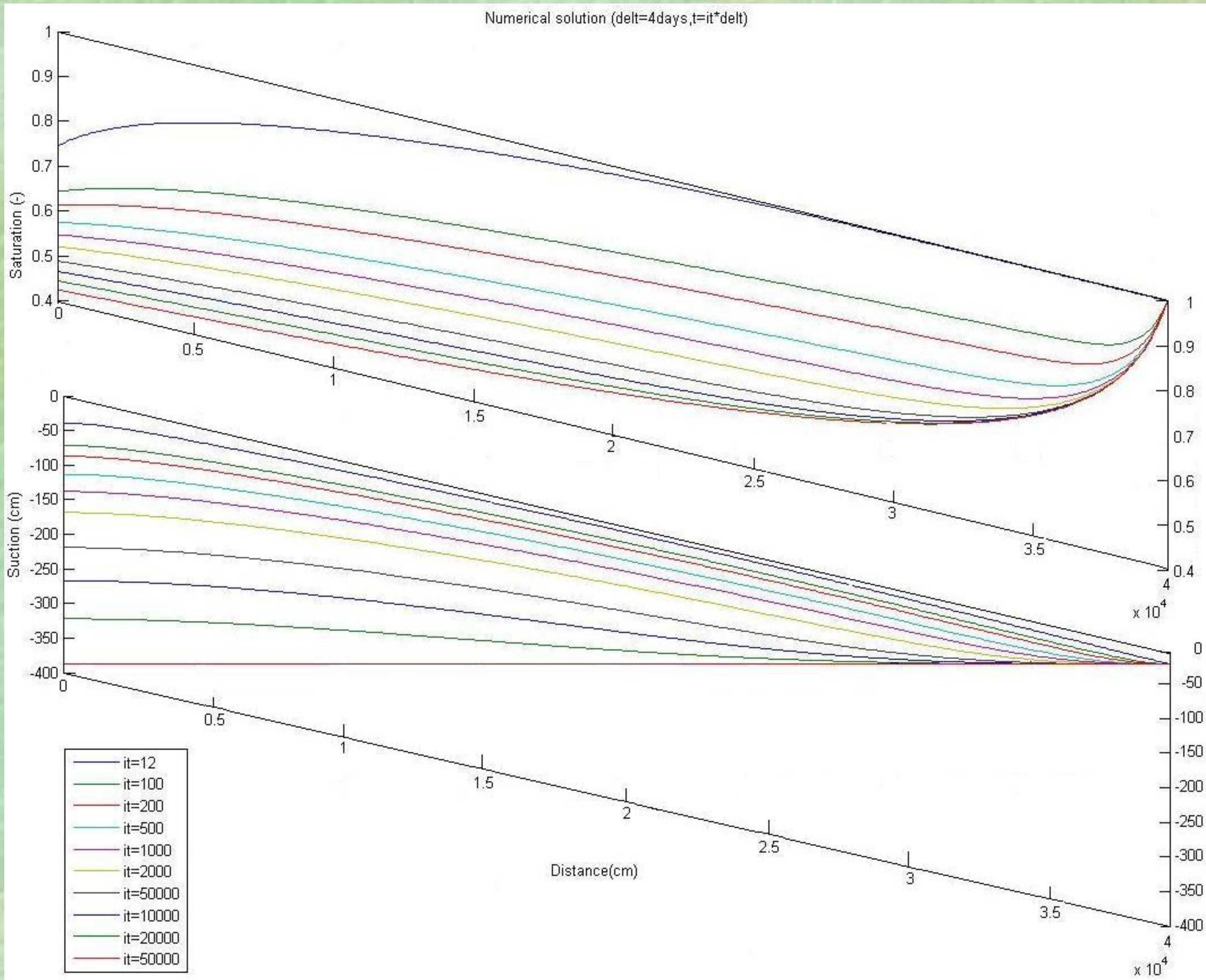


Figure 1) A sloped shallow aquifer


Numerical solution (delt=4days,t=it*delt)
Saturation (-)
Suction (cm)
Distance(cm)
x 10^4
it=12
it=100
it=200
it=500
it=1000
it=2000
it=50000
it=10000
it=20000
it=50000

Gravity Dominated Flow

- The scheme uses a power expression as before, except the time surrogate, Є(t), is used in order to satisfy the boundary conditions.

$$\psi_g = \left(\frac{x}{x_s} \cdot \frac{t_c}{t}\right)^{\frac{-b}{2b+2}}$$

- where ψ is the suction, *xs* locates the saturated portion of the aquifer, Є is a function of time, and *b* is a Clapp-Hornberger soil index.

$$\psi_g = \left(\frac{x + \epsilon}{x_s + \epsilon}\right)^{\frac{-b}{2b+2}\left(1 - \frac{x}{x_s}\right)^2}$$

- Flow from the aquifer is initially constant (blue zone – Figure 2) until a critical time *tc,* at which time capillary forces begin to have an effect. When this occurs, *xs* equals *L* and *tc,* can be determined as:

$$t_c = \frac{\int_0^L \phi(1-s)}{k_s \Lambda}$$

Suction Dominated Flow

- The flow during the next stage after tc (green zone – Figure 2) is initially driven by gravity forces. An increasing portion of the aquifer becomes inactive. Here the suction is sufficient to resist the elevation head. Thus, suction is given by:

$$\psi = \psi_0 - \Lambda(L - x)$$

Combined Flow

- It can be argued that the hydraulic resistance is proportional to the square of suction. Therefore, using a parallel electric circuit analog :

$$\psi = -\left(\frac{w}{\psi_g^2} + \frac{1-w}{\psi_f^2}\right)^{-\frac{1}{2}}$$

- *w* is a weighting factor which has a simple quadratic relation with Є .

- At a given time, the system is completely defined by Є and *w*. These can be determined from the conditions at the *x=o* boundary. The no flow boundary condition gives:

$$\left.\frac{\partial \psi}{\partial x}\right|_{x=0} = \Delta$$

- Applying the following mass balance at the x=0 boundary

$$-\varphi \frac{\partial s}{\partial \epsilon} \cdot \left.\frac{\partial \epsilon}{\partial t}\right|_{x=0} = k \left.\frac{\partial^2 \Psi}{\partial x^2}\right|_{x=0}$$

- yields both Є(t) and w(t) and ultimately Ψ(x,t) and s(x,t).

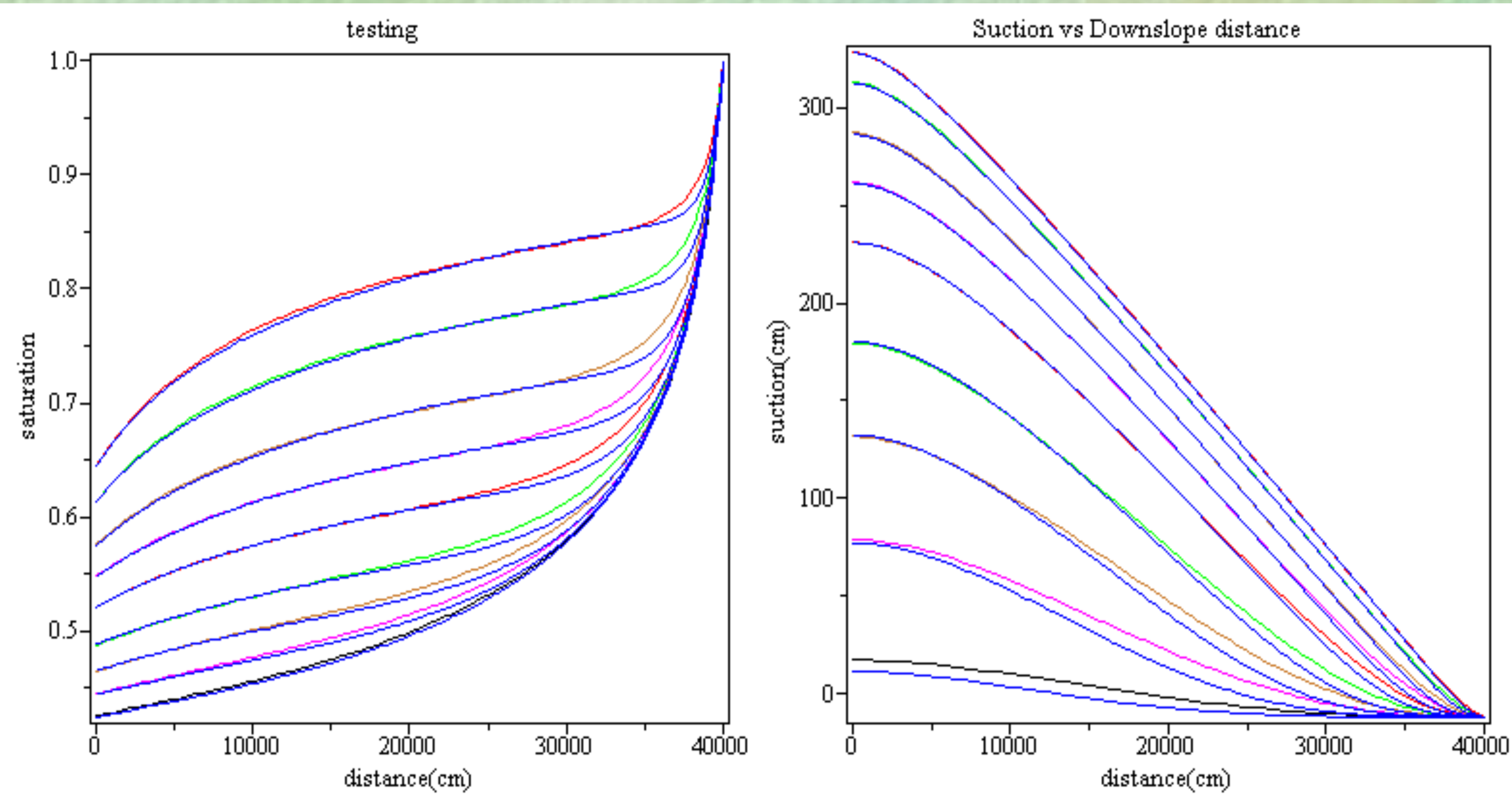
- **TEST RESULTS**
- Richards' equation is the governing equation for water flow in variably saturated soil, and here used as numerical "truth" to which the proposed solution is compared.

$$\eta \frac{\partial \psi}{\partial t} = \frac{\partial}{\partial z}\left(K \frac{\partial \psi}{\partial z} + K \right)$$

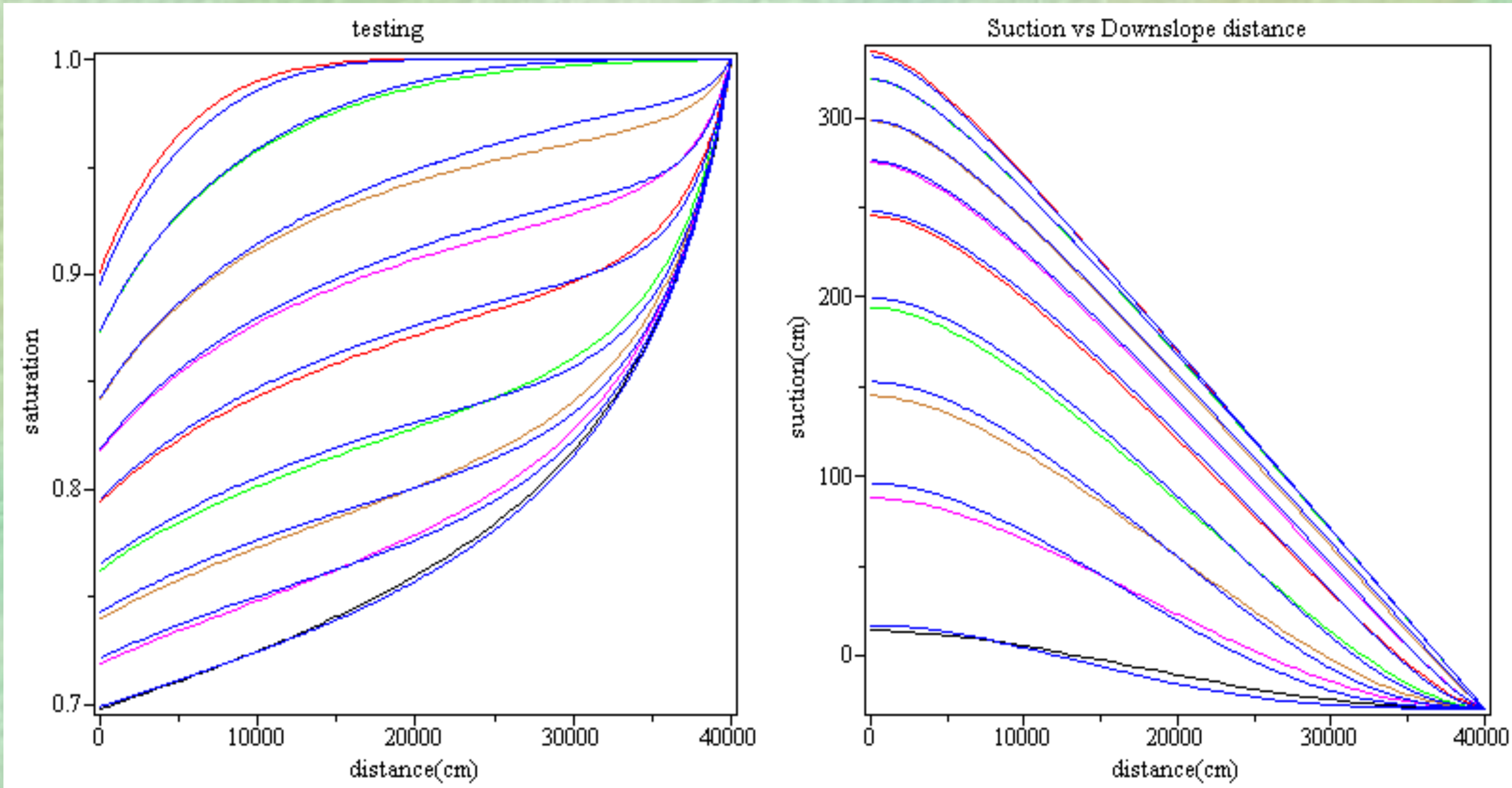
- where η is the specific storage. The numerical implementation has been validated against published simulation results.

- The analytical solution is tested by using a 400 m long aquifer, with slope of 0.01. The soil properties are defined by Clapp-Hornberger parameters for sand and for silt at opposite corners of the SCS soil triangle

- The differences between the analytical and numerical solutions are minor. The bulk saturation differences are less than 0.005 percent absolute.

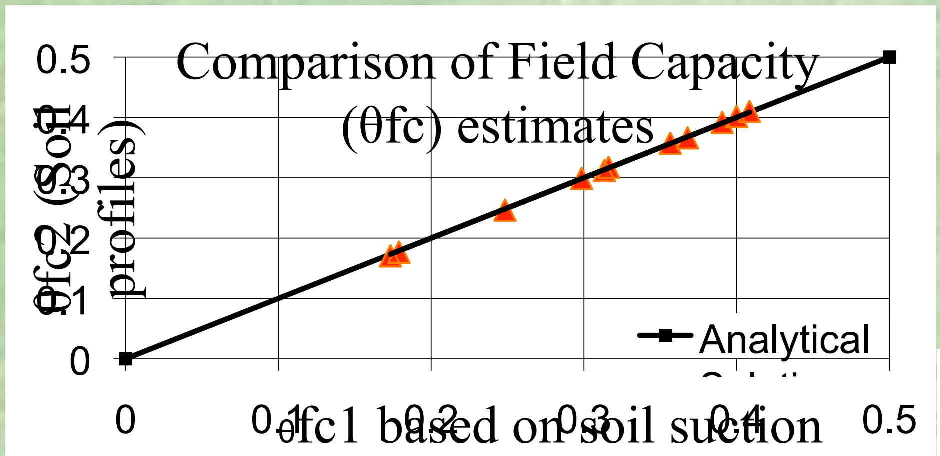


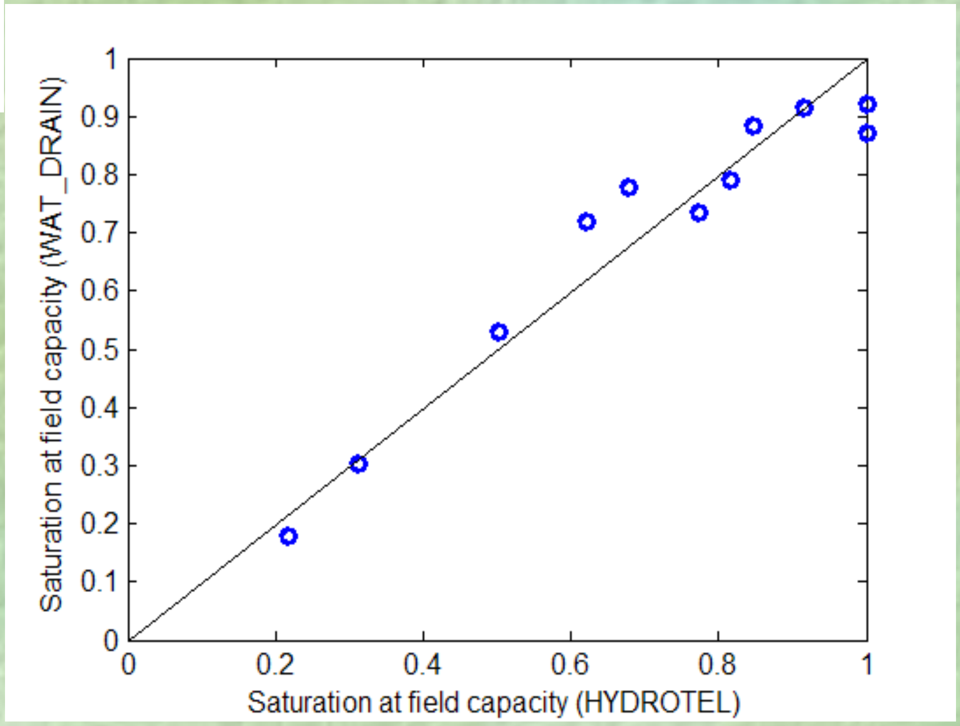
Soil profiles for sand at variable time (Left: saturation; Right: total potential)
Blue- numerical solution; Color-analytical solution



Soil profiles for silt at variable time (Left: saturation; Right: total potential) Blue-numerical solution; Color-analytical solution

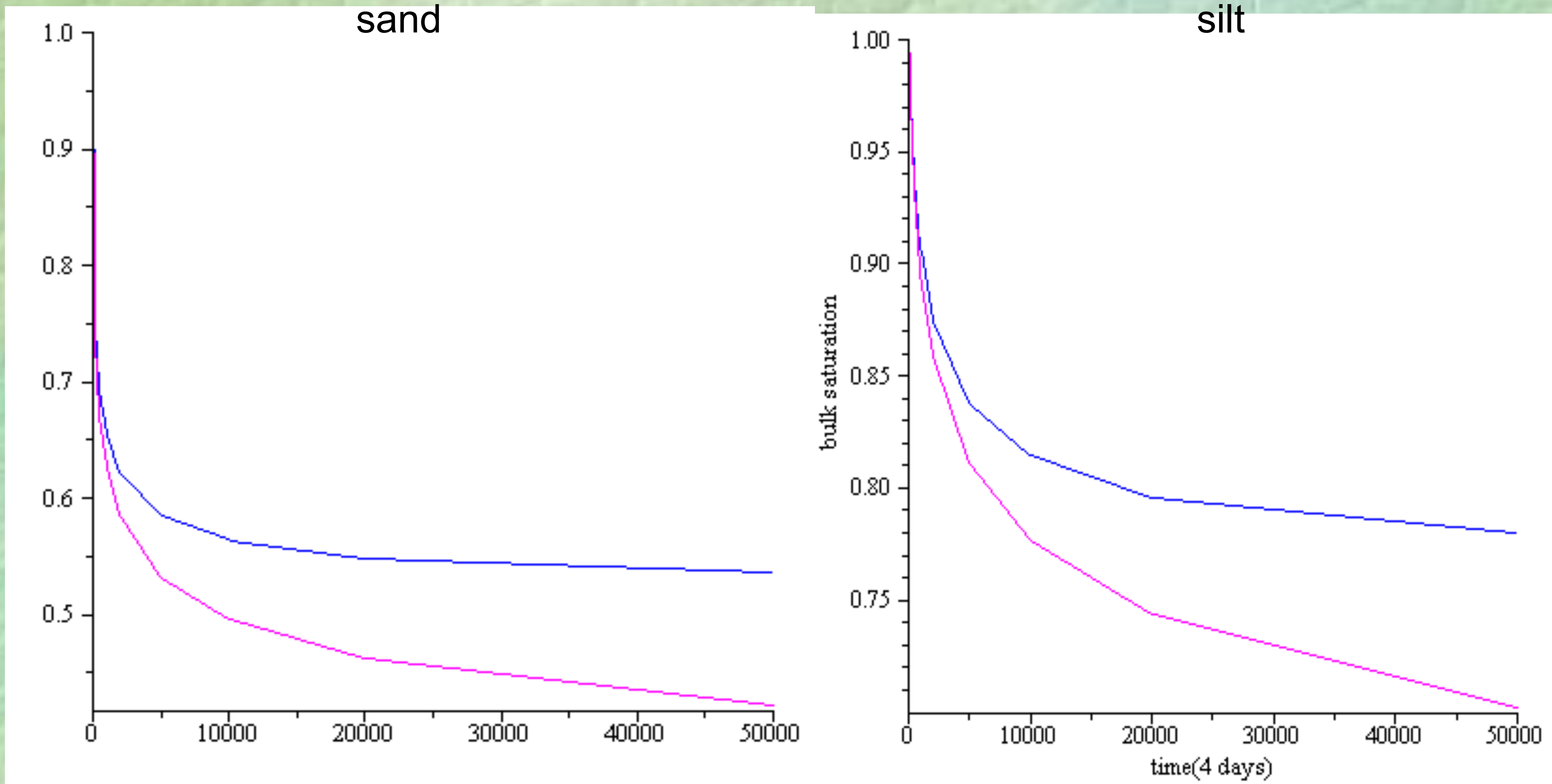
Field Capacity Comparison





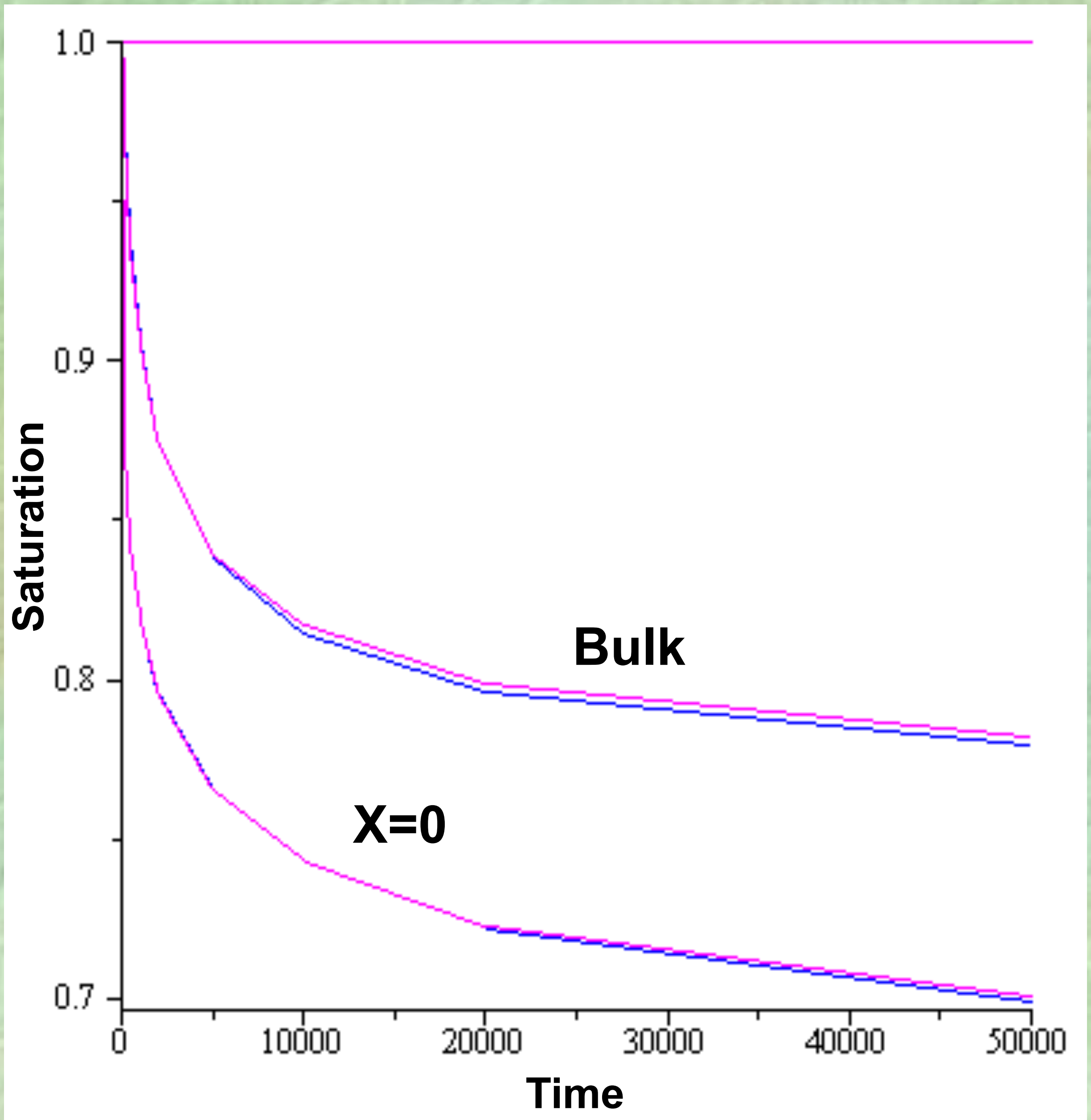
BULK SATURATION

Bulk saturation for the old and new methods is compared in Figure 5. The new method is expected to generate much shallower and prolonged recession curves than MESH currently produces. The blue line represents both the analytical and numerical solutions.



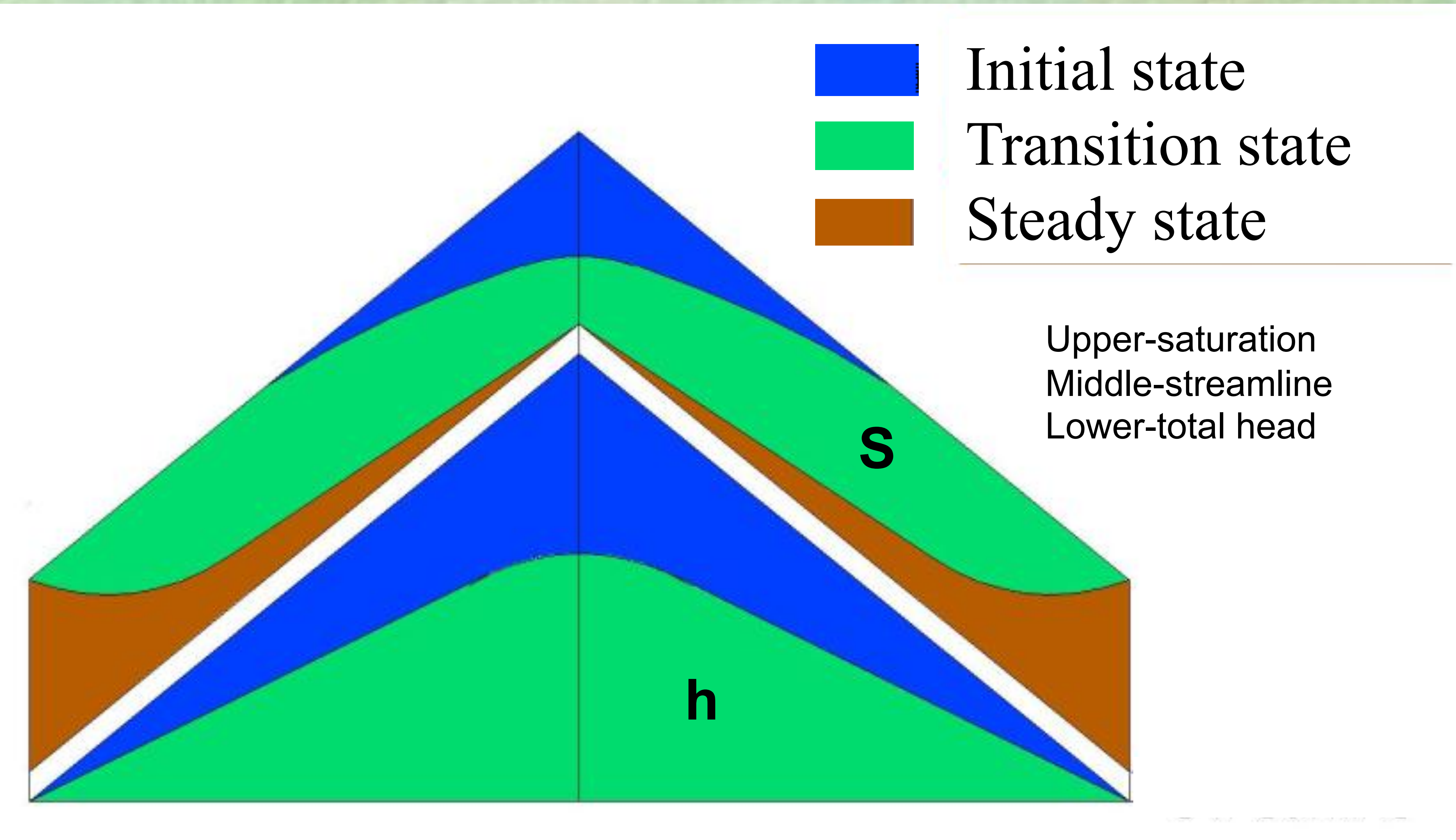
Bulk saturation without suction (pink), with suction (blue).

- **<u>CONCLUSION</u>**
- The new analytical scheme was evaluated against the numerical solution of the RE and is shown capable of reproducing the saturation and suction distributions for widely different soil types.
- The solution uses only Clapp-Hornberger parameters, slope and slope length all of which are available in MESH. Furthermore, the implementation algorithm will be the same as WATDRAIN.
- The solution is for a streamline which can be differential element for any soil layer with either constant or variable properties.
- The procedures are general, therefore suitable for CRHM or other distributed hydrological models as well as land surface schemes.



1.0
0.9
0.8
0.7
Saturation
Bulk
X=0
0
10000
20000
30000
40000
50000
Time

IP3 Tile


Initial state
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Upper-saturation
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Lower-total head
s
h